Janelas

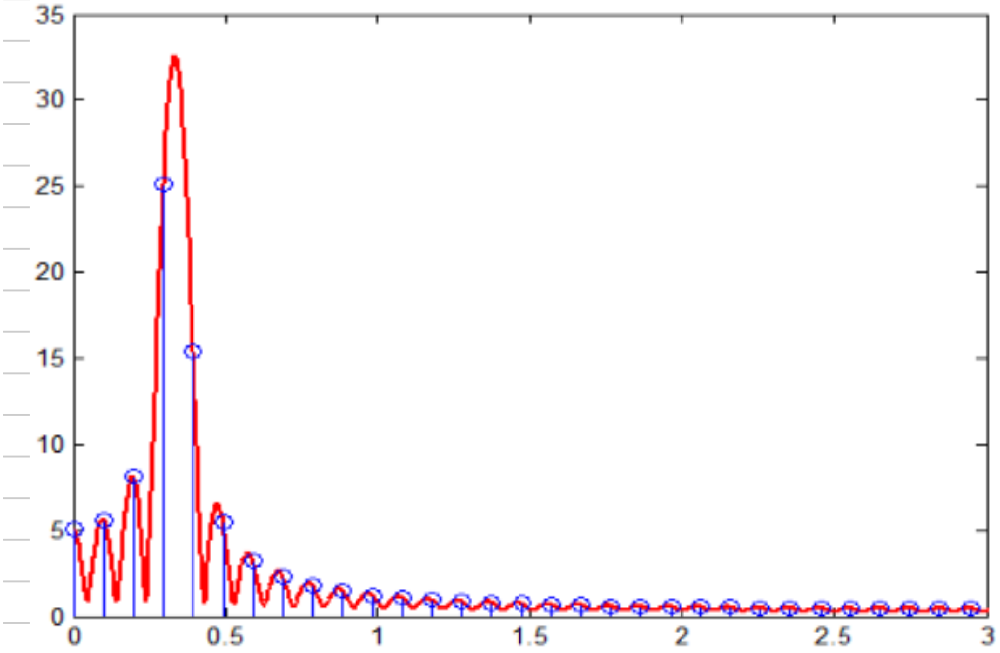
- Uso para melhorar vazamento
- Uso para projeto de filtro

Melhorando o vazamento: janelamento

Visão alternativa do vazamento.

Recordando: $X[k] = \sum_{n=0}^{N-1} x[n] e^{\mp j \frac{2\pi}{N} kn}$

$= X(e^{\mp j\omega}) \Big|_{\omega = \frac{2\pi}{N} k}$ se $x[n] = 0$ fora de $[0, N-1]$



$\Rightarrow$ DFT aproxima o espectro do sinal

$x_w[n] = x[n]\, r[n]$, $r[n] = \begin{cases} 1, & n = 0, \dots, N-1 \\ 0, & \text{c. c.} \end{cases}$

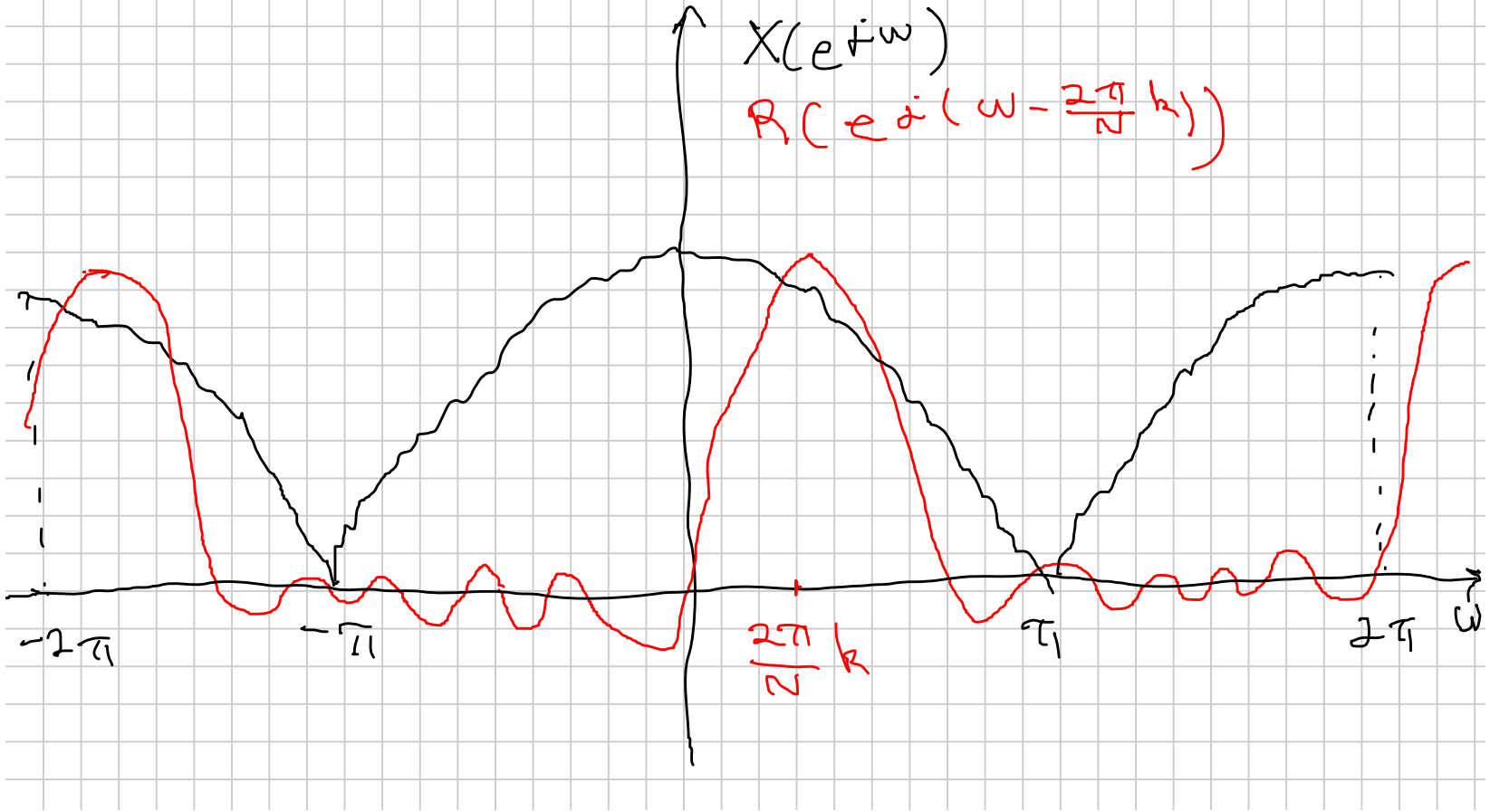
$$R(e^{j\omega}) = \sum_{n=0}^{N-1} e^{-j\omega n} = \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}} \quad \left(\text{PG com } N \text{ termos, razão } r = e^{-j\omega}\right)$$

$$R(e^{j\omega}) = \frac{e^{-j\omega\frac{N}{2}}}{e^{-j\omega/2}} \; \frac{e^{j\omega\frac{N}{2}} - e^{-j\omega N/2}}{e^{j\frac{\omega}{2}} - e^{-j\omega/2}}$$

$$= e^{-j\frac{\omega}{2}(N-1)} \; \frac{\operatorname{sen} \omega N/2}{\operatorname{sen} \omega/2}$$

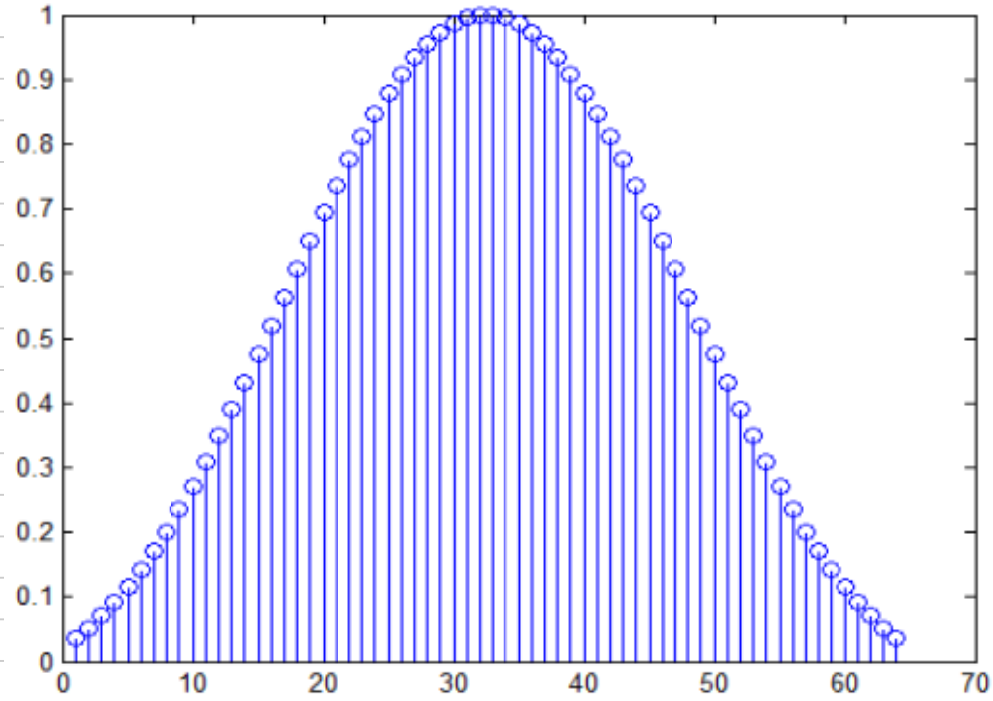
$\Rightarrow X_w(e^{j\omega}) = X(e^{j\omega}) * R(e^{j\omega})$

$$\Rightarrow X[k] = X_w(e^{j\omega})\Big|_{\omega=\frac{2\pi}{N}k} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})R(e^{j(\theta-\frac{2\pi}{N}k)})d\theta$$

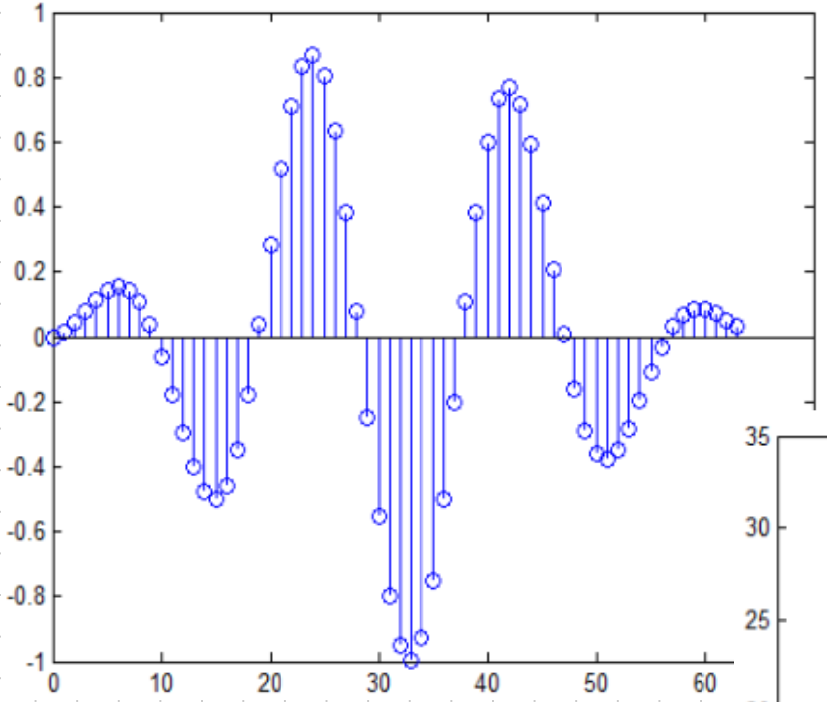


Janelas: multiplicar por algo diferente de r[n]

Exemplo: Kaiser

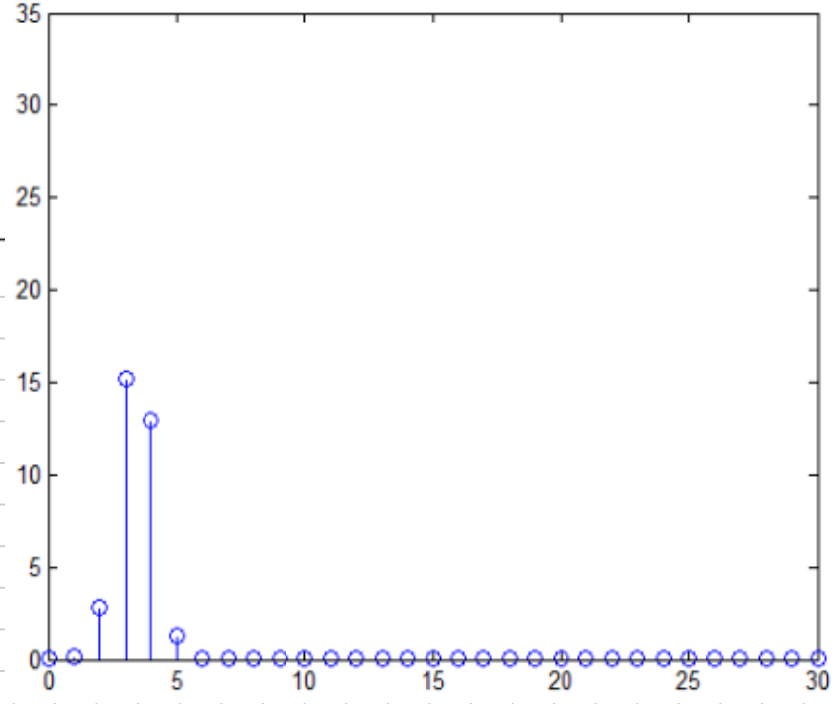


Transições menos abruptas nas bordas ⇒ menos altas frequências

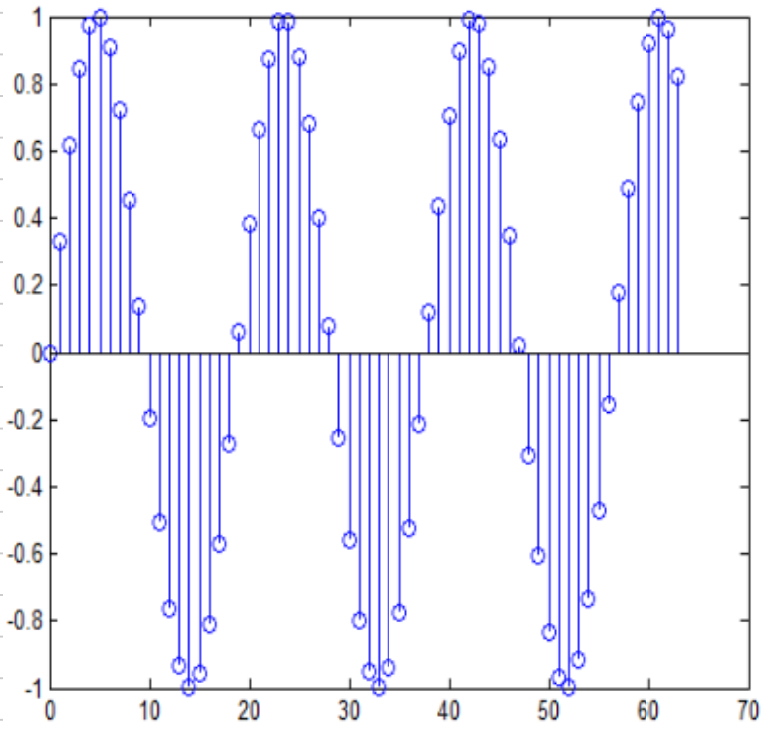


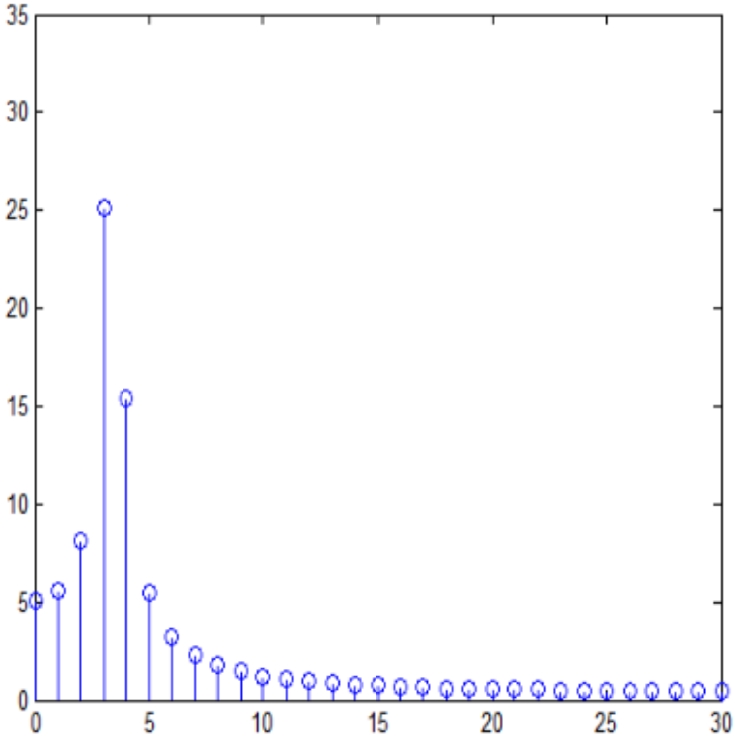
$\text{Sen}\left(\frac{2\pi}{64} 3.4n\right) \times \text{Kaiser}$

$|X[k]| \rightarrow$



Resultado com janela retangular

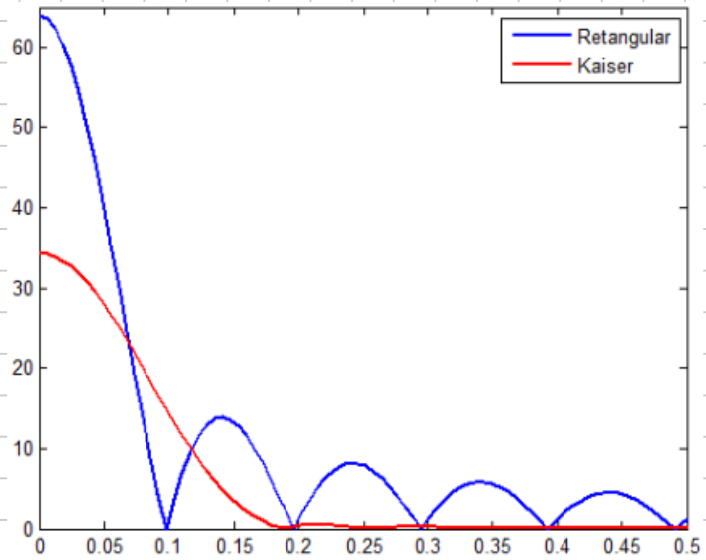




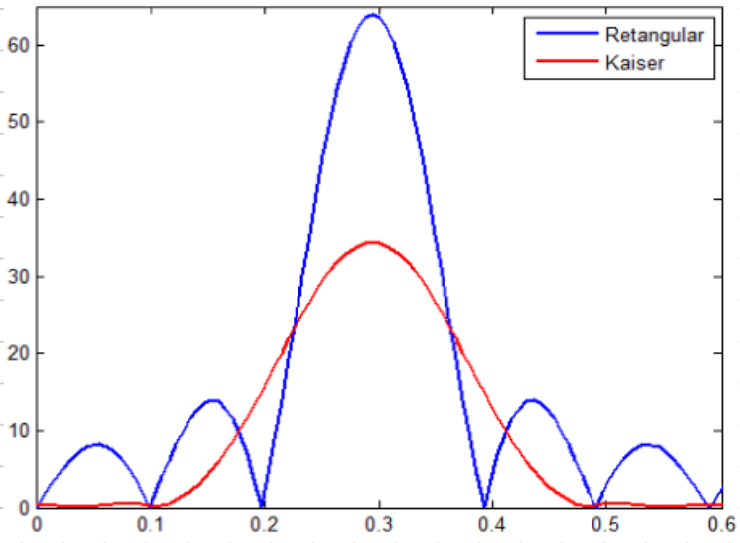
Janelas em frequência: DFT calcula

$$X[k] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, W\left(e^{j\left(\theta - \frac{2\pi}{N}k\right)}\right) d\theta$$

$k = 0$



$k = 3$



Lembre-se que existem freqs. negativas

Troca maior lóbulo central por uma menor influência dos freqs. mais distantes.