A STATE-SPACE APPROACH TO SEMI-BLIND SIGNAL DETECTION IN FAST FREQUENCY-SELECTIVE FADING MIMO CHANNELS

Murilo B. Loiola and Renato R. Lopes

Department of Communications - School of Electrical and Computer Engineering University of Campinas - UNICAMP - C.P 6101, 13083-852, Campinas, Brazil E-mails: {mloiola, rlopes}@decom.fee.unicamp.br

ABSTRACT

In this paper, we propose a semi-blind state-space based receiver that jointly performs channel estimation and data detection in MIMO systems subject to fast frequency-selective fading. To accomplish these two tasks, we first define state equations representing the dynamics of channel and transmitted signals. Then, we obtain the state vector by concatenating the transmitted signals and the channel coefficients. This choice of state vector leads to a nonlinear observation equation and hence to the use of the Extended Kalman Filter (EKF) to estimate the states variables. We then develop the EKF and show that the proposed receiver is a generalization of many similar receivers for SISO channels. We also develop a reduced complexity version of the proposed algorithm. Simulation results show the performance gains of the proposed receiver when compared to other commonly used receivers.

1. INTRODUCTION

Recently, multiple-input – multiple-output (MIMO) systems have attracted significant attention as a means of achieving high data rates in wireless communications. However, as data rate increases, the wireless channel starts causing considerable distortion to the transmitted signals by introducing intersymbol interference (ISI). In addition, due to the relative motion between transmitter and receiver, the fading channel is time-varying. The resulting time-varying frequency-selective fading channel is usually unknown to the receiver. This makes the detection of the transmitted symbols a difficult task. To overcome this difficulty, conventional receivers employ a channel estimator to track the channel and an equalizer to mitigate the ISI.

Among the most widely known approaches to channel tracking and equalization are state-space based receivers. An important characteristic of state-space receivers implemented by the well-known Kalman filter (KF) [1] is its inherent ability to deal with nonstationary environments. Examples of the

state-space formulation and the application of the Kalman filter in MIMO systems can be found in [2–5]. In [2] a Kalman filter that uses the outputs of a minimum mean square error (MMSE) decision-feedback equalizer (DFE) is developed to track Rician MIMO frequency-selective channels. Channel estimation using Kalman filters for MIMO-OFDM systems is studied in [3, 4], while a Kalman turbo equalizer for quasistatic MIMO channels is proposed in [5].

One problem of the usual schemes of separated channel estimation and data detection is that, since the estimator and equalizer use the estimates obtained from each other, the correlation between the estimates of the channel and data symbols can be significant [6]. Generally, this correlation is very difficult to quantify and is usually ignored in the equalization or detection process. Joint channel estimation and data detection schemes, on the other hand, do not present this problem and usually have better performance.

An optimal approach to the problem of joint channel estimation and data detection is to perform joint maximum likelihood (ML) estimation. The joint ML solution is obtained through an exhaustive search procedure, in which channel estimation is performed for each possible candidate data sequence. This approach, however, usually presents high computational complexity, growing exponentially with the length of the channel and the constellation size. For this reason, we propose in this paper a state-space receiver that jointly performs channel estimation and data detection in MIMO systems subject to fast frequency-selective fading. The proposed receiver has a computational complexity that does not grow with the constellation size and that is smaller than that of optimum joint ML scheme. As we will see in the following sections, the joint formulation leads to a nonlinear observation equation, preventing the use of the standard KF. Hence, we use the extended Kalman filter (EKF) [1] to compute recursive estimates of the state variables.

Schemes similar to the one presented in this paper can be found in [6–10]. Single-input/single-output (SISO) receivers using the EKF to estimate the channel and equalize the received signals are developed in [6–8]. As shown in a later section, we can consider that the algorithms presented in [6,7] are

We acknowledge the financial support received from FAPESP (05/55310-8) and CNPq (311844/2006-5).

special cases of the method proposed here. In [9], the EKF is employed to joint estimate the channel and the state transition matrix, while channel and frequency offset in MIMO-OFDM systems are jointly estimated in [10].

The rest of this paper is organized as follows: the system model is presented in section 2. In section 3, we detail the proposed receiver while in section 4, we present some simulation results. Finally, section 5 concludes the paper.

2. SYSTEM MODEL

We consider a system with N_T transmit antennas sending independent signals to N_R receive antennas through a timevarying MIMO channel with ISI and length L. The relationship between transmitted and received signals in a time instant k can then be written as

$$\mathbf{r}_{k} = \sum_{l=0}^{L-1} \mathbf{H}_{l,k} \mathbf{x}_{k-l} + \mathbf{n}_{k}, \qquad (1)$$

where $\mathbf{r}_k = [r_k^{(1)} \cdots r_k^{(N_R)}]^{\mathrm{T}}$ is the vector containing the signals observed in each receive antenna, $\mathbf{x}_k = [x_k^{(1)} \cdots x_k^{(N_T)}]^{\mathrm{T}}$ is the vector of transmitted symbols, $\mathbf{H}_{l,k}, l = 0, \ldots, L-1$ represents the $N_R \times N_T$ matrix coefficients of MIMO channel impulse response, $\mathbf{n}_k = [n_k^{(1)} \cdots n_k^{(N_R)}]^{\mathrm{T}}$ contains samples of additive, zero-mean, complex white gaussian noise with covariance matrix $\mathbf{R} = \sigma_n^2 \mathbf{I}_{N_R}$, and \mathbf{I}_J is the identity matrix of order J.

If we stack N successive received vectors in a new observation vector $\tilde{\mathbf{r}}_k = [\mathbf{r}_k^{\mathrm{T}} \mathbf{r}_{k-1}^{\mathrm{T}} \cdots \mathbf{r}_{k-N+1}^{\mathrm{T}}]^{\mathrm{T}}$, it is possible to rewrite (1) as

$$\tilde{\mathbf{r}}_k = \mathcal{H}_k \tilde{\mathbf{x}}_k + \tilde{\mathbf{n}}_k, \qquad (2)$$

with $\tilde{\mathbf{x}}_k$ denoting the $N_T(N + L - 1)$ vector of concatenated transmitted symbols, $\tilde{\mathbf{x}}_k = [\mathbf{x}_k^T \mathbf{x}_{k-1}^T \cdots \mathbf{x}_{k-N-L+2}^T]^T$, $\tilde{\mathbf{n}}_k = [\mathbf{n}_k^T \mathbf{n}_{k-1}^T \cdots \mathbf{n}_{k-N+1}^T]^T$ is the length $N_R N$ noise vector and \mathcal{H} is an $N_R N \times N_T(N + L - 1)$ block Toeplitz matrix of channel coefficients given by

$$\mathcal{H}_{k} = \begin{bmatrix} \mathbf{H}_{0,k} & \mathbf{H}_{1,k} & \cdots & \mathbf{H}_{L-1,k} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{0,k} & \cdots & \mathbf{H}_{L-2,k} & \mathbf{H}_{L-1,k} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}_{0,k} & \cdots & \cdots & \mathbf{H}_{L-1,k} \end{bmatrix} .$$

$$(3)$$

According to the widely used wide-sense stationary uncorrelated scattering (WSSUS) model [11], the channel coefficients are independent, zero-mean, complex gaussian random variables with time autocorrelation function [2, 5–7, 12]

$$\mathbf{E}\left[h_{l,k}^{(i,j)}\left(h_{l,t}^{(i,j)}\right)^{*}\right] \approx \mathcal{J}_{0}(2\pi f_{D}T \left|k-t\right|), \qquad (4)$$

where $h_{l,k}^{(i,j)}$, $i = 1, ..., N_R$, $j = 1, ..., N_T$, l = 0, ..., L-1is the (i, j) element of matrix $\mathbf{H}_{l,k}$, \mathcal{J}_0 is the zero-order Bessel function of the first kind, $f_D T$ is the normalized Doppler rate (assumed the same for all transmit-receive antenna pairs) and T is the baud duration. Hence, the normalized spectrum for each channel tap is expressed as

$$S(f) = \begin{cases} \frac{1}{\pi f_D T} \frac{1}{\sqrt{1 - \left(\frac{f}{f_D T}\right)^2}}, & |f| < f_D T\\ 0, & \text{otherwise.} \end{cases}$$
(5)

Although exact modeling of channel dynamics by finite length autoregressive (AR) processes is impossible because the time autocorrelation function (4) is nonrational and the spectrum (5) is bandlimited, we can approximate the time evolution of channel coefficients by low-order AR processes. This is possible because the first few correlation terms of (4), for small lags, capture most of the channel dynamics [2, 6]. Therefore, following [2, 6, 7, 12], we herein approximate the MIMO channel variations by a first order AR process. Thus, defining the length LN_RN_T channel vector \mathbf{h}_k by stacking all columns of the channel matrices $\mathbf{H}_{l,k}$, $l = 0, \ldots, L - 1$, i.e.,

$$\mathbf{h}_{k} = \begin{bmatrix} h_{0,k}^{(1,1)} & h_{0,k}^{(2,1)} & \cdots & h_{0,k}^{(N_{R},1)} & h_{0,k}^{(1,2)} & \cdots & h_{0,k}^{(1,N_{T})} \\ \cdots & h_{0,k}^{(N_{R},N_{T})} & h_{1,k}^{(1,1)} & \cdots & h_{L-1,k}^{(1,1)} & \cdots & h_{L-1,k}^{(N_{R},N_{T})} \end{bmatrix}^{\mathrm{T}},$$

the time evolution of the channel is given by

$$\mathbf{h}_k = \mathbf{F}_1 \mathbf{h}_{k-1} + \mathbf{w}_k, \tag{6}$$

where

$$\mathbf{F}_1 = \beta \mathbf{I}_{LN_B N_T},\tag{7}$$

 $\beta = \mathcal{J}_0(2\pi f_D T)$, \mathbf{w}_k is a vector of length LN_RN_T containing independent samples of circularly symmetric, zeromean, gaussian excitation noise with covariance matrix $\mathbf{W} = \sigma_w^2 \mathbf{I}_{LN_RN_T}$, and $\sigma_w^2 = (1 - |\beta|^2)P_k$, with $P_k = \mathbb{E}\left[|h_k^{(m)}|^2\right]$, $m = 1, \ldots, LN_RN_T$.

The speed of channel variations, quantified by β in (7), is determined by the Doppler shift or, equivalently, by the relative velocity between the N_T transmit and the N_R receive antennas. The greater the value of $f_D T$, the smaller the value of β , leading to faster channel variations. The magnitude of these variations is controlled by σ_w . It is worth mentioning that the model (6) can characterize time invariant ($f_D T = 0$) and time-varying ($f_D T > 0$), as well as frequency flat (L = 1) and frequency selective (L > 1) MIMO channels.

It is also necessary to model the dynamics of the stacked transmitted symbols vector $\tilde{\mathbf{x}}_k$. Observing that $\tilde{\mathbf{x}}_k$ has a time-shifting structure, its time evolution can be described by a Markov-like process, where the current stacked transmitted symbols vector is formed by shifting the previous one and adding the current transmitted vector \mathbf{x}_k . Mathematically, this is accomplished by writing

$$\tilde{\mathbf{x}}_k = \mathbf{F}_2 \tilde{\mathbf{x}}_{k-1} + \mathbf{u}_k,\tag{8}$$

where the shift matrix \mathbf{F}_2 and the excitation noise \mathbf{u}_k are defined, respectively, as

$$\mathbf{F}_{2} = \begin{bmatrix} \mathbf{0}_{N_{T} \times N_{T}(N+L-2)} & \mathbf{0}_{N_{T} \times N_{T}} \\ \mathbf{I}_{N_{T}(N+L-2)} & \mathbf{0}_{N_{T}(N+L-2) \times N_{T}} \end{bmatrix}$$
(9)

and

$$\mathbf{u}_{k} = \begin{bmatrix} \mathbf{x}_{k}^{\mathrm{T}} & \mathbf{0}_{1 \times N_{T}(N+L-2)} \end{bmatrix}^{\mathrm{T}}.$$
 (10)

The covariance matrix of \mathbf{u}_k equals

$$\mathbf{U} = \mathbf{E} \begin{bmatrix} \mathbf{u}_{k} \mathbf{u}_{k}^{\mathrm{H}} \end{bmatrix}$$
$$= \sigma_{u}^{2} \begin{bmatrix} \mathbf{I}_{N_{T}} & \mathbf{0}_{N_{T} \times N_{T}}(N+L-2) \\ \mathbf{0}_{N_{T}}(N+L-2) \times N_{T} & \mathbf{0}_{N_{T}}(N+L-2) \times N_{T}(N+L-2) \end{bmatrix}$$
(11)

and $\mathbf{0}_{i \times j}$ denotes an *i*-by-*j* matrix of zeros. Notice that, as \mathbf{u}_k is formed by symbols of a discrete alphabet, it is not gaussian.

3. PROPOSED RECEIVER

In order to develop a state-space receiver, we must first define the state vector. Since we wish to jointly estimate the channel coefficients and the transmitted signals, an obvious choice for the state vector \mathbf{s}_k is

$$\mathbf{s}_{k} = \begin{bmatrix} \tilde{\mathbf{x}}_{k}^{\mathrm{T}} & \mathbf{h}_{k}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}.$$
 (12)

Once the state vector s_k is defined, we have to model the dynamics of the state variables. Assuming that the transmitted symbols are independent from the channel coefficients, it is possible to write the state transition matrix from time k - 1 to time k, $\mathbf{F}_{k,k-1}$, as a block diagonal matrix of the form

$$\mathbf{F}_{k,k-1} = \begin{bmatrix} \mathbf{F}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_1 \end{bmatrix}, \qquad (13)$$

where \mathbf{F}_2 is given by (9) and \mathbf{F}_1 by (7).

Therefore, combining (2), (6), (8), (12) and (13), we obtain the state-space description of the problem of joint channel estimation and signal detection in MIMO systems:

$$\mathbf{s}_k = \mathbf{F}_{k,k-1}\mathbf{s}_{k-1} + \mathbf{q}_k, \qquad (14a)$$

$$\tilde{\mathbf{r}}_k = \mathbf{C}(k, \mathbf{s}_k) + \tilde{\mathbf{n}}_k,$$
 (14b)

with

$$\mathbf{q}_{k} = \begin{bmatrix} \mathbf{u}_{k}^{\mathrm{T}} & \mathbf{w}_{k}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{Q} = \mathrm{E}\begin{bmatrix} \mathbf{q}_{k} \mathbf{q}_{k}^{\mathrm{H}} \end{bmatrix} = \begin{bmatrix} \mathbf{U} & \mathbf{0} \\ \mathbf{0} & \mathbf{W} \end{bmatrix},$$
(15)

and

$$\mathbf{C}\left(k,\mathbf{s}_{k}\right)=\mathcal{H}_{k}\tilde{\mathbf{x}}_{k}.$$
(16)

Due to the choice of state vector made in (12), the observation equation (14b) becomes nonlinear on the state variables. This occurs because the received vector, $\tilde{\mathbf{r}}_k$ in (14b), depends now on the product between state variables, as can be clearly seen from (16). Consequently, it is not possible to directly apply the classical KF to estimate the state vector. Instead, we can use the EKF to obtain recursive estimates of state variables. The EKF, in fact, applies the standard KF to a nonlinear system by linearizing the nonlinear functions about the estimated state vector [1].

To accomplish the linearization of the nonlinear observation function $\mathbf{C}(k, \mathbf{s}_k) = \mathcal{H}_k \tilde{\mathbf{x}}_k$, we have to calculate the Jacobian of $\mathbf{C}(k, \mathbf{s}_k)$ about $\hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}}$, i.e., about the estimate of \mathbf{s}_k , in the time instant k, based on the data observed from the beginning to the time k - 1. The Jacobian is denoted by

$$\left[\mathbf{C}_{k}\right]_{i,j} = \left.\frac{\partial \left[\mathbf{C}\left(k,\mathbf{s}_{k}\right)\right]_{i}}{\partial \left[\mathbf{s}_{k}\right]_{j}}\right|_{\mathbf{s}_{k} = \hat{\mathbf{s}}_{k}|\mathbf{r}_{1:k-1}}, \quad (17)$$

where the element (i, j) of matrix \mathbf{C}_k is obtained by differentiating the *i*th row of $\mathcal{H}_k \tilde{\mathbf{x}}_k$ with respect to the state variable *j*. Carrying out the computations in (17), the Jacobian \mathbf{C}_k can be expressed as

$$\mathbf{C}_{k} = \frac{\partial(\mathcal{H}_{k}\tilde{\mathbf{x}}_{k})}{\partial \mathbf{s}_{k}}\Big|_{\mathbf{s}_{k} = \hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}}} = \begin{bmatrix} \mathbf{a}_{k} \otimes \mathbf{I}_{N_{R}} \\ \mathbf{a}_{k-1} \otimes \mathbf{I}_{N_{R}} \\ \mathbf{a}_{k-1} \otimes \mathbf{I}_{N_{R}} \\ \mathbf{a}_{k-N+1} \otimes \mathbf{I}_{N_{R}} \end{bmatrix},$$
(18)

where $\mathbf{a}_k = [\hat{\mathbf{x}}_k^{\mathrm{T}} \cdots \hat{\mathbf{x}}_{k-L+1}^{\mathrm{T}}]$ and \otimes represents the Kronecker product. It is important to notice that \mathbf{a}_k and $\hat{\mathcal{H}}_k$ are formed, respectively, from estimates of $\tilde{\mathbf{x}}_k$ and \mathbf{h}_k contained in $\hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}}$.

The expression (18) is quite general. In fact, the expressions derived for SISO systems in [6,7] can be viewed as particular cases of (18) when $N_T = N_R = 1$.

Using (12)–(16) and (18), it is now possible to employ the EKF to recursively estimate the transmitted symbols and track the channel. The EKF is shown in the table of algorithm 3.1, where $\mathbf{K}_{k,k-1}$ is defined as

$$\mathbf{K}_{k,k-1} = \mathbf{E}\left[(\mathbf{s}_k - \hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}}) (\mathbf{s}_k - \hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}})^{\mathrm{H}} \right].$$
(19)

Algorithm 3.1 Extended Kalman Filter (EKF)

Prediction step

$$\hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}} = \mathbf{F}_{k,k-1}\hat{\mathbf{s}}_{k-1|\mathbf{r}_{1:k-1}}$$
(20a)

$$\mathbf{K}_{k,k-1} = \mathbf{F}_{k,k-1}\mathbf{K}_{k-1,k-1}\mathbf{F}_{k,k-1}^{\mathrm{H}} + \mathbf{Q}_{k} \qquad (20b)$$

Filtering step

$$\mathbf{G}_{k} = \mathbf{K}_{k,k-1} \mathbf{C}_{k}^{\mathrm{H}} \left[\mathbf{C}_{k} \mathbf{K}_{k,k-1} \mathbf{C}_{k}^{\mathrm{H}} + \mathbf{R}_{k} \right]^{-1}$$
(21a)

$$\alpha_k = \tilde{\mathbf{r}}_k - \mathbf{C}\left(k, \hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}}\right) \tag{21b}$$

$$\hat{\mathbf{s}}_{k|\mathbf{r}_{1:k}} = \hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}} + \mathbf{G}_k \alpha_k \tag{21c}$$

$$\mathbf{K}_{k,k} = \left[\mathbf{I} - \mathbf{G}_k \mathbf{C}_k\right] \mathbf{K}_{k,k-1}$$
(21d)

If we further develop (20a) and (20b) by using (7), (9), (12), (13) and (15), the computational complexity of EKF can

be reduced since the matrix multiplications can be replaced by

$$\hat{\mathbf{s}}_{k|\mathbf{r}_{1:k-1}} = \mathbf{F}_{k,k-1} \hat{\mathbf{s}}_{k-1|\mathbf{r}_{1:k-1}} = \begin{bmatrix} \mathbf{0}_{N_T \times 1}^{\mathrm{T}} & \hat{\mathbf{x}}_{k-1}^{\mathrm{T}} & \cdots & \hat{\mathbf{x}}_{k-N-L+2}^{\mathrm{T}} & \beta \hat{\mathbf{h}}_{k-1}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
(22)

and

$$\mathbf{K}_{k,k-1} = \begin{bmatrix} \mathbf{I}_{N_{T}} & \mathbf{0}_{N_{T} \times N_{T}} (N \pm L - 2) \pm \underline{L} N_{R} N_{T} \\ \mathbf{0}_{N_{T}(N+L-2)} + L N_{R} N_{T} \times N_{T} & \mathbf{A}_{1} & \mathbf{B}_{1} \\ \mathbf{C}_{1} & \mathbf{D}_{1} \end{bmatrix},$$
(23)

where

$$\begin{aligned} \mathbf{A}_{1} &= \mathbf{K}_{k-1,k-1} \left(1 : \mathbf{P}_{1}, 1 : \mathbf{P}_{1} \right), \\ \mathbf{B}_{1} &= \beta \mathbf{K}_{k-1,k-1} \left(1 : \mathbf{P}_{1}, \mathbf{P}_{2} : \mathbf{P}_{3} \right), \\ \mathbf{C}_{1} &= \beta \mathbf{K}_{k-1,k-1} \left(\mathbf{P}_{2} : \mathbf{P}_{3}, 1 : \mathbf{P}_{1} \right), \\ \mathbf{D}_{1} &= \beta^{2} \mathbf{K}_{k-1,k-1} \left(\mathbf{P}_{2} : \mathbf{P}_{3}, \mathbf{P}_{2} : \mathbf{P}_{3} \right) + \sigma_{w}^{2} \mathbf{I}_{LN_{P}N_{T}}, \end{aligned}$$

 $P_1 = N_T(N + L - 2), P_2 = N_T(N + L - 1) + 1, P_3 = N_T(N + L - 1) + LN_RN_T$, and the notation $\mathbf{K}_{k-1,k-1}(i: j, k:l)$ follows the MATLAB notation and represents a submatrix of matrix $\mathbf{K}_{k-1,k-1}$ formed by taking its rows *i* to *j* and columns *k* to *l*.

Recalling from the state vector definition (12) that the estimated stacked transmitted signals vector is of the form

$$\hat{\mathbf{\hat{x}}}_{k} = \begin{bmatrix} \hat{\mathbf{x}}_{k}^{\mathrm{T}} & \hat{\mathbf{x}}_{k-1}^{\mathrm{T}} & \cdots & \hat{\mathbf{x}}_{k-N-L+2}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \qquad (24)$$

we find that $\hat{\mathbf{s}}_{k|\mathbf{r}_{1:k}}$ contains information not only about the currently transmitted symbol vector $\hat{\mathbf{x}}_k$ but also about the symbol vectors corresponding to the previous N + L - 1 time instants. Hence, the final estimated vector $\hat{\mathbf{x}}_k$ at instant k is obtained by averaging the corresponding transmitted symbol estimates contained in state estimates from time k to time k+N+L-2. The detected symbol vector is then obtained by passing this final averaged symbol vector through a decision device.

4. SIMULATION RESULTS

In this section, simulation results are presented to illustrate the performance of the proposed semi-blind, reduced complexity, state-space receiver. For comparison purposes, the performance of a receiver composed by an LMS channel estimator and a Kalman equalizer (referred to as LMS+KF receiver in this section), operating iteratively, is also shown.

We consider a baseband digital communication system with 2 transmit antennas sending 3×10^6 independent, 4-QAM, symbols to 4 receive antennas. Each data block is formed by 25 training symbols followed by 125 information symbols. As we focus on semi-blind schemes, both EKF and LMS+KF receivers continue updating the channel estimates after the end of the training sequence. Specifically, for the LMS+KF



Fig. 1. Mean square error of channel tracking.

receiver, the LMS estimator first estimates the channel using the training sequence. When the information symbols are transmitted, the LMS channel estimator works in a decisiondirected mode, using the symbol estimates generated by the Kalman equalizer.

We simulate two MIMO frequency-selective channels with L = 2 taps of equal average power of 0 dB. In the first scenario, we use a normalized Doppler rate $f_D T = 0.001$ while in the second, we use $f_D T = 0.01$. The observation vector $\tilde{\mathbf{r}}_k$ is formed by stacking N = 6 received vectors when $f_D T = 0.001$ and N = 10 received vectors when $f_D T = 0.01$. It is also assumed that the receivers know the noise variance σ_n^2 . The results presented in this section correspond to an average of ten channels realizations. Also, simulation results indicate that, on average, the EKF implemented with (22) and (23) performs 10% faster than the EKF using (20a) and (20b).

The mean square error (MSE) of channel tracking for both receivers, as a function of SNR per receive antenna, is shown in Fig. 1. We clearly see that the LMS estimator, with a step size of 0.015, is not able to track the channel variations. In fact, the MSE of the LMS channel estimator is almost constant for the whole SNR range considered. The proposed receiver, on the other hand, presents an almost linear decrease of MSE with the SNR. For an SNR greater than 20 dB, the EKF shows similar MSE for both Doppler rates, indicating that it can track fast channel variations.

Figure 2 presents the symbol error rates (SER) of EKF and LMS+KF in both simulation scenarios. We also plot the SER of a Kalman equalizer with perfect knowledge of channel state information (KF-CSI), i.e., perfect channel knowledge. As expected, all receivers have better performance for slower channel variations (smaller f_DT). It can be observed that the performance of the proposed joint receiver is superior to that of LMS+KF. For an SER of 10^{-3} , the difference between the



Fig. 2. Symbol error rate of different receivers.

EKF and the KF-CSI is about 6 dB for $f_D T = 0.001$ and 9 dB for $f_D T = 0.01$. The LMS+KF, on the other hand, does not reach a SER smaller than 10^{-1} . This can be explained by the poor channel estimates provided by the LMS estimator. With these channel estimates, the Kalman equalizer is not able to correctly detect the transmitted symbols. The erroneous detected symbols are fed back to the LMS estimator, degenerating the new channel estimates.

5. CONCLUSIONS

In this paper, we propose a reduced complexity state-space, semi-blind method to jointly perform channel estimation and data detection in MIMO systems. In the developed statespace formulation, the observation equation is nonlinear since we assume that transmitted symbols and channel coefficients are state variables. This nonlinearity leads us to the use of extended Kalman filter to obtain recursive estimates of the state variables. One advantage of the proposed receiver over other joint schemes is that the computational complexity of the EKF does not grow with the constellation size. Simulation results indicate that the joint receiver presented in this paper is able to track fast channel variations. The results also illustrate the superiority of EKF over a receiver using an LMS channel estimator and a Kalman equalizer.

6. ACKNOWLEDGEMENTS

The authors are indebted to Prof. Romis Attux and Dr. Ricardo Suyama, both from UNICAMP, for their great insight into the problem formulation.

7. REFERENCES

[1] T. Kailath, A. H. Sayed, and B. Hassibi, *Linear Estimation*, Prentice Hall, 2000.

- [2] C. Komninakis, C. Fragouli, A. H. Sayed, and R. D. Wesel, "Multi-Input Multi-Output Fading Channel Tracking and Equalization Using Kalman Estimation," *IEEE Transactions on Signal Processing*, vol. 50, no. 5, pp. 1065–1076, May 2002.
- [3] R. J. Piechocki, A. R. Nix, J. P. McGeehan, and S. M. D. Armour, "Joint Blind and Semi-Blind Detection and Channel Estimation for Space-Time Trellis Coded Modulation Over Fast Faded Channels," *IEE Proceedings on Communications*, vol. 150, no. 6, pp. 419–426, December 2003.
- [4] M. Enescu, T. Roman, and V. Koivunen, "State-Space Approach to Spatially Correlated MIMO OFDM Channel Estimation," *Signal Processing*, vol. 87, no. 1, pp. 2272–2279, 2007.
- [5] S. Roy and T. M. Duman, "Soft Input Soft Output Kalman Equalizer for MIMO Frequency Selective Fading Channels," *IEEE Transactions on Wireless Communications*, vol. 6, no. 2, pp. 506–514, February 2007.
- [6] X. Li and T. F. Wong, "Turbo Equalization with Nonlinear Kalman Filtering for Time-Varying Frequency-Selective Fading Channels," *IEEE Transactions on Wireless Communications*, vol. 6, no. 2, pp. 691–700, February 2007.
- [7] H. Gerlach, D. Dahlhaus, M. Pesce, and W. Xu, "Joint Kalman Channel Estimation and Equalization for the UMTS FDD Downlink," in *Proc. IEEE 2003 Vehicular Technology Conference*, VTC2003, 2003, vol. 2, pp. 1263–1267.
- [8] K. J. Kim and R. A. Iltis, "Joint Detection and Channel Estimation Algorithms for QS-CDMA Signals Over Time-Varying Channels," *IEEE Transactions on Communications*, vol. 50, no. 5, pp. 845–855, May 2002.
- [9] D. Schafhuber, G. Matz, and F. Hlawatsch, "Kalman Tracking of Time-Varying Channels in Wireless MIMO-OFDM Systems," in *Proceedings of the 37th Asilomar Conference on Signals, Systems, and Computers*, 2003, vol. 2, pp. 1261–1265.
- [10] T. Roman, Advanced Receiver Structures for Mobile MIMO Multicarrier Communication Systems, PhD. Thesis, Helsinki University of Technology, Espoo, Finland, April 2006.
- [11] W. C. Jakes, *Microwave Mobile Communications*, John Wiley and Sons, New York, 1974.
- [12] E. Karami and M. Shiva, "Blind Multi-Input Multi-Output Channel Tracking Using Decision-Directed Maximum-Likelihood Estimation," *IEEE Transactions* on Vehicular Technology, vol. 56, no. 3, pp. 1447–1454, May 2007.