A New Approach for Joint Channel Estimation and Data Detection in MIMO Wireless Systems

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***Abstract*— In this paper, we use the *cubature Kalman filter* (CKF) for joint channel estimation and data detection in multiple-input multiple-output (MIMO) systems subject to fast frequency-selective fading. First, based on a model of the channel coefficients and transmitted symbol dynamic, the problem is formulated in the state-space form, leading to a nonlinear observation equation. Then, the CKF is presented and some changes are proposed to improve the estimation and detection processes in the receiver, and to overcome modeling errors. Finally, through numerical simulations, it is shown that the CKF can be used to track channel variations and detect transmitted symbols with low error rates.**

I. Introduction

The use of multiple transmission and/or reception antennas has played a central role in the design of modern wireless communication systems [1], [2]. In practice, when using high data transmission rates in these systems, the transmitted symbols are subject to inter-symbol interference (ISI). Furthermore, if there exists a relative movement between the receiver and the transmitter, the wireless channel through which data is transmitted will vary in time. Therefore, in order to implement practical wireless systems that use multiple transmit and/or receive antennas, it is essential to devise methods to correctly detect symbols transmitted under such conditions.

Indeed, to mitigate channel effects, the receiver must employ estimators for channel tracking and equalizers for ISI compensation. Usually, the estimators and equalizers work separately, and the correlation between estimated channel and data symbols introduced by using estimates from detection in channel estimation process and, conversely, by using channel estimates to detect the symbols, is ignored. An interesting alternative to this usual approach is described in [3], where a joint semi-blind detection and channel estimation is proposed using Bayesian estimation theory [4]. However, although a Bayesian filter provides an optimal analytical solution to non-linear filtering problems, its solution involves a great computational cost. Therefore, approximations to the Bayesian paradigm are necessary in order to develop practical implementations. In [5] the *cubature Kalman filter* (CKF) was proposed as a Gaussian approximation for the Bayesian filter, providing accurate estimates and being able to solve a wide range of nonlinear problems.

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Within this context, in this paper we aim to efficiently perform a joint channel estimation and data detection in multiple-input multiple-output (MIMO) systems subject to fast frequency-selective fading. In order to achieve this goal, we first model the channel estimation and data detection problem in the state-space form, obtaining a single augmented state-equation and a non-linear observation equation. Since in our state-space formulation the process equation is linear, it is possible to develop a hybrid receiver using the well-known *Kalman filter* approach to perform the prediction updates, and using the CKF to perform the measurement updates. Therefore, using the proposed state-space model, we derive a filter based on the CKF that takes into account properties of the system in order to obtain a filtering algorithm with maximum computational efficiency. Also, in order to compensate for a possible modeling mismatch of the channel dynamics, we introduce an aging factor, following the Recursive Fading Memory Filtering theory [6], that progressively reduces the importance of past channel estimates in the filtering algorithm. Finally, we analyze the behavior of our filter in doubly-selective channel scenarios through numerical simulations, comparing its performance with the *extended Kalman filter* (EKF) [7], which is a commonly used approximation to the Bayesian Filter [8].

The remainder of this paper is organized as follows: Section II describes the system model and the state-space formulation. The *cubature Kalman filter* derivation for the considered problem is presented in Section III. Section IV reports simulation results and, finally, Section V concludes the paper.

II. System Model

We consider a MIMO system with N_T transmitting antennas and N_R receiving antennas interfering with each other. Due to ISI, a signal received by an antenna i is also subject to interference from symbols transmitted previously. The relationship between received and transmitted signals at time k can be expressed as:

$$\mathbf{y}_k = \sum_{l=0}^{L-1} \mathbf{H}_{l,k}\mathbf{x}_{k-l} + \mathbf{n}_k \tag{1}$$

$$= \begin{bmatrix} \mathbf{H}_{0,k} & \mathbf{H}_{1,k} & \dots & \mathbf{H}_{L-1,k} \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_{k-l} \\ \vdots \\ \mathbf{x}_{k-L+1} \end{bmatrix} + \mathbf{n}_k$$

where L is the number of channel taps, $\mathbf{y}_k = \begin{bmatrix} y_{1,k} & y_{2,k} & \dots & y_{N_R,k} \end{bmatrix}^T$ is the vector of signals received by the N_R antennas, $\mathbf{x}_k = \begin{bmatrix} x_{1,k} & x_{2,k} & \dots & x_{N_T,k} \end{bmatrix}^T$ is the vector of signals transmitted by the N_T antennas, $\mathbf{H}_{l,k}$ is a $N_R \times N_T$ matrix, $\forall l = 0, \dots, L-1$, whose element on the $i-th$ row and $j-th$ column, denoted by $h_{i,j,l,k}$, corresponds to the value of the $l-th$ channel coefficient between the $j-th$ transmitting antenna and the $i-th$ receiving antenna at time k, and $\mathbf{n}_k = \begin{bmatrix} n_{1,k} & n_{2,k} & \dots & n_{N_R,k} \end{bmatrix}^T$ is a vector of white, Gaussian, circularly symmetric, zero mean, i.i.d noise samples, with variance equal to σ_n^2.

In addition, in order to represent the time variation of the MIMO channel taps, we assume the typical wide-sense stationary uncorrelated scattering (WSSUS) model [9]. In this model, the channel taps have time-autocorrelation properties that are governed by the Doppler rate and are given by

$$E[h_{i,j,l,k} h^*_{i,j,l,k+\Delta k}] \approx \mathscr{J}_0(2\pi f_D T_s |\Delta k|) \tag{2}$$

where $\mathscr{J}_0$ is the zero-order Bessel function of the first kind, $f_D T_s$ is the normalized Doppler rate and T_s is the baud duration.

We consider that the channel coefficients remain constant during $N \geq 1$ symbols and follow the time-autocorrelation function (2) for time evolution between blocks of N symbols. Thus, we can stack the N received vectors, and write the received signals as a linear combination of the transmitted symbols, obtaining

$$\tilde{\mathbf{y}}_k = \mathscr{H} \tilde{\mathbf{x}}_k + \tilde{\mathbf{n}}_k \tag{3}$$

where $\tilde{\mathbf{y}}_k = \begin{bmatrix} \mathbf{y}_k^T & \mathbf{y}_{k-1}^T & \dots & \mathbf{y}_{k-N+1}^T \end{bmatrix}^T$ is a column vector containing the N received vectors, $\tilde{\mathbf{x}}_k = \begin{bmatrix} \mathbf{x}_k^T & \mathbf{x}_{k-1}^T & \dots & \mathbf{x}_{k-N-L-2}^T \end{bmatrix}^T$ is a vector containing $N + L - 1$ stacked transmitted vectors, $\tilde{\mathbf{n}}_k = \begin{bmatrix} \mathbf{n}_k^T & \mathbf{n}_{k-1}^T & \dots & \mathbf{n}_{k-N-L-2}^T \end{bmatrix}^T$ is the noise vector and

$$\mathscr{H} = \begin{bmatrix} \mathbf{H}_{0,k} & \dots & \mathbf{H}_{L-1,k} & \dots & 0 \\ 0 & \mathbf{H}_{0,k} & \dots & \mathbf{H}_{L-1,k} & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \mathbf{H}_{0,k} & \dots & \mathbf{H}_{L-1,k} \end{bmatrix}$$

is a block Toepletiz matrix with each block representing the channel coefficients.

Also, defining $\mathrm{vec}(\cdot)$ as the operator that stacks the columns of a matrix on top of each other, the column vector

$$\mathbf{h}_k = \mathrm{vec}(\begin{bmatrix} \mathbf{H}_{0,k} & \mathbf{H}_{1,k} & \dots & \mathbf{H}_{L-1,k} \end{bmatrix})$$

represents the channel coefficients. As described in [10], we can approximate the channel dynamics by a first order autoregressive process (AR), and its time evolution can be written as

$$\mathbf{h}_k = \beta \mathbf{h}_{k-1} + \mathbf{w}_k \tag{4}$$

where $\beta = \mathscr{J}_0(2\pi f_D T_s)$, $\mathbf{w}_k$ is a vector of white, Gaussian, circularly symmetric, zero mean noise samples, with covariance matrix $\mathbf{W} = \sigma_w^2 I_{N_R N_T}$, and $\sigma_w^2 = (1 - |\beta|^2)$.

Finally, (3) can be rewritten as a combination of the channel coefficients as follows:

$$\tilde{\mathbf{y}}_k = \mathscr{X} \mathbf{h}_k + \tilde{\mathbf{n}}_k \tag{5}$$

where

$$\mathscr{X} = \begin{bmatrix} \mathbf{x}_k^T & \mathbf{x}_{k-1}^T & \dots & \mathbf{x}_{k-L+1}^T \\ \mathbf{x}_{k-1}^T & \mathbf{x}_{k-2}^T & \dots & \mathbf{x}_{k-L}^T \\ \vdots & \vdots & \dots & \vdots \\ \mathbf{x}_{k-N+1}^T & \mathbf{x}_{k-N}^T & \dots & \mathbf{x}_{k-N-L+2}^T \end{bmatrix} \otimes I_{N_R}$$

and $\otimes$ denotes the Kronecker product. Note that (4) and (5) suggest the formulation of a filtering problem to perform the channel tracking as in [11]. However, in a practical communication system, some elements in $\mathscr{X}$ may not be known by the receiver, preventing a linear state-space modeling.

In this paper, we treat both the channel coefficients and the transmitted symbols as variables to be estimated. Thus, to develop the joint estimation and detection state model, the augmented state vector, $\mathbf{z}_k$, is defined as

$$\mathbf{z}_k = \begin{bmatrix} \tilde{\mathbf{x}}_k^T & \mathbf{h}_k^T \end{bmatrix}^T. \tag{6}$$

Observe that in order to obtain the augmented state-equation, it is necessary to describe the dynamic behavior of the vector $\tilde{\mathbf{x}}_k$, which contains the stacked transmitted symbols. To this end, note that, as time evolves, new transmitted symbols are added to the top of $\tilde{\mathbf{x}}_k$, while existing symbols are shifted towards the bottom. Consequently, defining $\mathbf{0}_{i \times j}$ as an i-by-j matrix of zeros, this time-shifting structure can be modeled as

$$\tilde{\mathbf{x}}_k = \mathbf{F}_x \tilde{\mathbf{x}}_{k-1} + \mathbf{u}_k, \tag{7}$$

where

$$\mathbf{F}_x = \begin{bmatrix} \mathbf{0}_{N_T \times N_T(N+L-2)} & \mathbf{0}_{N_T \times N_T} \\ \mathbf{I}_{N_T(N+L-2)} & \mathbf{0}_{N_T(N+L-2) \times N_T} \end{bmatrix}$$

is a shift matrix and

$$\mathbf{u}_k = \begin{bmatrix} \mathbf{x}_k^T & \mathbf{0}_{1 \times N_T(N+L-2)} \end{bmatrix}^T$$

is a non-Gaussian noise with covariance matrix given by

$$\begin{aligned} \mathbf{U} &= E[\mathbf{u}_k \mathbf{u}_k^H] \\ &= \sigma_u^2 \begin{bmatrix} \mathbf{I}_{N_T} & \mathbf{0}_{N_T \times N_T(N+L-2)} \\ \mathbf{0}_{N_T(N+L-2) \times N_T} & \mathbf{0}_{N_T(N+L-2) \times N_T(N+L-2)} \end{bmatrix}. \end{aligned}$$

With the dynamic behavior of $\tilde{\mathbf{x}}_k$ and $\mathbf{h}_k$ in hand, (given by (7) and (4), respectively), it is possible to write the state transition matrix from time $k-1$ to time k as

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_h \end{bmatrix} \tag{8}$$

where $\mathbf{F}_h = \beta \mathbf{I}_{L N_R N_T}$.

Therefore, using (5)–(8), we define the process and observation equations for the problem of joint channel estimation and data detection as

$$\mathbf{z}_k = \mathbf{F} \mathbf{z}_{k-1} + \mathbf{q}_k \tag{9}$$

$$\tilde{\mathbf{y}}_k = \mathscr{X}\mathbf{h}_k + \tilde{\mathbf{n}}_k = \mathscr{H}\tilde{\mathbf{x}}_k + \tilde{\mathbf{n}}_k \tag{10}$$

where

$$\mathbf{q}_k = \begin{bmatrix}\mathbf{u}_k^T & \mathbf{w}_k^T\end{bmatrix}^T,$$
$$\mathbf{Q} = E[\mathbf{q}_k\mathbf{q}_k^H] = \begin{bmatrix}\mathbf{U} & \mathbf{0}\\ \mathbf{0} & \mathbf{W}\end{bmatrix}.$$

Note that the process equation described in (9) is linear and allows us to estimate the predicted state ($\hat{\mathbf{z}}_{k|k-1}$) and the predicted error covariance ($P_{k|k-1}$) using Kalman filtering theory [12]. The observation equation, on the other hand, is a nonlinear function of the state variables, since it involves a multiplication between state vector elements. Consequently, for measurement updates, the Kalman filter can not be used and the optimal solution can only be achieved through Bayesian filtering.

III. Proposed Solution

A. Nonlinear Treatment

The Bayesian filter [4] provides an optimal analytic solution to nonlinear filtering problems. However, it requires the evaluation of several intricate multidimensional integrals, preventing its implementation in practical systems. In this sense, the challenge lays in developing a sub-optimal solution to nonlinear filtering problems that provides accurate estimation without great computational cost.

The *extended Kalman filter* (EKF) is the commonly used algorithm to approach nonlinear filtering problems [7], employing first-order Taylor series to approximate the nonlinear state-space equations. But, due to its first-order linearization, the EKF presents divergence problems when the initial estimative is inaccurate, or when modeling mismatches are present. For these reasons, the EKF does not work well in the majority of practical environments.

With the motivation of obtaining an accurate nonlinear filter that could be applied to solve a wide range of nonlinear filtering problems, the *cubature Kalman filter* (CKF), presented in [5], was recently proposed. The CKF approximates the Bayesian filter's statistical description by assuming that the conditional densities are Gaussian. Consequently, the Bayesian filter's solution can be reduced to the evaluation of Gaussian integrals weighted by known nonlinear functions, which allows the application of efficient numerical integration methods called "Cubature Rules"[13].

Finally, according to these rules, it is possible to write integrals of the form *nonlinear function* $\times$ *Gaussian* as follows:

$$\int_{\mathscr{R}^n} \mathbf{f}(\mathbf{x})\mathscr{N}(\mathbf{x};\mathbf{0},\mathbf{I})d\mathbf{x} \approx \sum_{i=1}^{m} \omega_i \mathbf{f}(\xi_i)$$

where ξ_i is the $i-th$ element of the set:

$$\xi_i = \sqrt{\frac{m}{2}}\left\{\begin{pmatrix}1\\ \vdots\\ 0\end{pmatrix},\ldots,\begin{pmatrix}0\\ \vdots\\ 1\end{pmatrix},\begin{pmatrix}-1\\ \vdots\\ 0\end{pmatrix},\ldots,\begin{pmatrix}0\\ \vdots\\ -1\end{pmatrix}\right\}$$

and

$$\omega_i = \frac{1}{m}, i = 1,2,\ldots,m = 2n.$$

As seen in [5], the CKF provides accurate results with reasonable complexity for nonlinear state-space estimation problems. Thus, we will use the CKF for the joint channel estimation and data detection problem. However, in order to improve the detection of the transmitted symbols and the estimation of the channel coefficients, we also implemented two changes to the CKF, as described in the sequel.

B. Fixed Lag Smoothing

Note that the estimated stacked vectors

$$\hat{\hat{\mathbf{x}}}_k = \begin{bmatrix}\hat{\mathbf{x}}_k^T & \hat{\mathbf{x}}_{k-1}^T & \ldots & \hat{\mathbf{x}}_{k-N-L-2}^T\end{bmatrix}^T$$

contains the estimation of the current transmitted symbols vector and the estimation of vectors transmitted in the $N+L-2$ past time instants. Thus, at time instant k, the CKF provides an estimate of $x_{k-N-L-2}$ based on all the observations up to time k. This estimate is better than that obtained at time instant $k-N-L-2$, since it is based on more information [7]. Thus, a fixed delay in detection is introduced, and the final estimate for vector $\hat{\mathbf{x}}_k$ is obtained at time $k+N+L-2$, where a decision device gives the detected symbols.

C. Recursive Fading Memory Filter

As already mentioned, we used a first-order autoregressive process to model the dynamic behavior of the MIMO channel coefficients. However, in practical systems the channel coefficients evolution does not follow this assumption, generating modeling mismatch which may degrade the estimator performance.

To compensate for modeling errors, we employed the *recursive fading memory filter* theory [7] in the proposed algorithm. These filters reduce the importance of the process equation in the estimation process by basing the estimate of the state on more recent measurements. Consequently, the resulting filter becomes less sensitive to modeling mismatches.

Recalling that the filter's estimative minimizes the error function J_N

$$J_N = \sum_{k=1}^{N}\left[(\mathbf{y}_k - \mathscr{X}\mathbf{h}_k)^H \mathbf{R}_n^{-1}(\mathbf{y}_k - \mathscr{X}\mathbf{h}_k) + \mathbf{w}_k^H \mathbf{Q}'^{-1}\mathbf{w}_k^H\right],$$

it is possible to increase the weight of the recent measurements by inserting an aging factor $\alpha > 1$, which provides an exponential weighting of the measured signals:

$$\hat{J}_N = \sum_{k=1}^{N}\left[(\mathbf{y}_k - \mathscr{X}\mathbf{h}_k)^H \alpha^{2k}\mathbf{R}^{-1}(\mathbf{y}_k - \mathscr{X}\mathbf{h}_k) + \mathbf{w}_k^H \alpha^{2k+2}\mathbf{Q}'^{-1}\mathbf{w}_k^H\right].$$

Thus, the noise covariance matrixes of the process and measurement equations can be redefined, respectively, as $\alpha^{2k}\mathbf{R}$ and $\alpha^{2k+2}\mathbf{Q}$, requiring only the insertion of a multiplicative factor α^2 in the algorithm error covariance matrix update:

$$\mathbf{P}_{k|k-1} = (\alpha\beta)^2 P_{k|k-1} + \sigma_w^2\mathbf{R}.$$

D. Summary

The proposed algorithm is able to efficiently approach the joint estimation and data detection problem. In addition, due to the problem structure, it is also possible to reduce its computational complexity. The algorithm's steps are shown in Table 1.

IV. Numerical Results

In this section, we report some numerical results to illustrate the proposed algorithm performance. We analyze its superior estimation ability even in critical situations, when the channel coefficient's variation is extremely fast, and we also demonstrate its behavior in face of modeling errors. In addition, for comparison purposes, the performance of a receiver using the *extended Kalman filter* (EKF) for joint estimation and detection is also shown.

First, we set a scenario in which the channel coefficients were generated by an autoregressive process as in (4), with 0 dB average power, and we assume that the receiver knows the noise variance σ_n^2. In this environment, a communication link is established between 2 transmitting and 4 receiving antennas, and 1×10^6 QPSK symbols are sent in frames composed by 25 training symbols and 125 data symbols. During the training period, the proposed algorithm works as a Kalman filter performing the estimation of the channel coefficients. During the data transmission period, the unknown symbols are jointly detected with the channel coefficients using the algorithm in Table 1. A normalized Doppler rate of $f_D T = 0.01$ is considered, and $N = 10$ symbol vectors were stacked in the receiver.

Fig. 1 presents the MSE of the channel estimation performed by the CKF, the EKF, and by an estimator with complete knowledge of all symbols, representing the best possible channel estimative (Best Estimation - BE) for a normalized doppler rate of 0.01. It is clear that the estimation provided by the CKF is better than the estimation provided by the EKF, being closer to the best possible channel estimative, which indicates the ability of the CKF to track the variation of channel coefficients. Note that for higher SNR's, the EKF error performance stops improving, while the MSE values for CKF continues to decrease. This can be explained mainly by the fact that very accurate measurements in some nonlinear filtering problems may result in numeric instability, which ends up interfering on channel estimation. Thus, besides being a better estimator, this result shows that the *cubature Kalman filter* is also less prone to numerical instabilities.

TABLE I: CKF with Aging Factor

Temporal Update
$\hat{\mathbf{z}}_{k\|k-1} = \left[\mathbf{0}^T_{N_T \times 1} \quad \hat{\hat{\mathbf{x}}}^T_{k-1\|k-1}(1:N_T(N+L-2)) \quad \beta\hat{\mathbf{h}}^T_{k-1\|k-1}\right]^T$
$\mathbf{A}_1 = P_{k-1\|k-1}(1:N_T(N+L-2), 1:N_T(N+L-2))$
$\mathbf{B}_1 = \alpha\beta P_{k-1\|k-1}(1:N_T(N+L-2), N_T(N+L-1):N_T(N+L-1)+LN_RN_T)$
$\mathbf{C}_1 = \alpha\beta P_{k-1\|k-1}(N_T(N+L-1):N_T(N+L-1)+LN_RN_T, 1:N_T(N+L-2))$
$\mathbf{D}_1 = \alpha^2\beta^2 P_{k-1\|k-1}(N_T(N+L-1):N_T(N+L-1)+LN_RN_T, N_T(N+L-1):N_T(N+L-1)+LN_RN_T) + \sigma_w^2\mathbf{I}$
$S_{k\|k-1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sqrt{\begin{matrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D}_1 \end{matrix}} \end{bmatrix}$
Measurement Update
$\mathbf{Z}_{i,k\|k-1} = (S_{k\|k-1}\xi_i + \hat{\mathbf{z}}_{k\|k-1})_{(i=1,\ldots,m)}$
$\mathbf{Y}_{i,k\|k-1} = \mathbf{h}(\mathbf{Z}_{i,k\|k-1})$
$\hat{\mathbf{y}}_{k\|k-1} = \frac{1}{m}\sum_{i=1}^m \mathbf{Y}_{i,k\|k-1}$
$P_{yy,k\|k-1} = \frac{1}{m}\sum_{i=1}^m \mathbf{Y}_{i,k\|k-1}\mathbf{Y}^T_{i,k\|k-1} - \hat{\mathbf{y}}_{k\|k-1}\hat{\mathbf{y}}^T_{k\|k-1} + R_k$
$P_{zy,k\|k-1} = \frac{1}{m}\sum_{i=1}^m \omega_i \mathbf{Z}_{i,k\|k-1}\mathbf{Y}^T_{i,k\|k-1} - \hat{\mathbf{z}}_{k\|k-1}\hat{\mathbf{y}}^T_{k\|k-1}$
$\mathbf{W}_k = P_{zy,k\|k-1}P_{yy,k\|k-1}{-1}$
$\hat{\mathbf{z}}_{k\|k} = \hat{\mathbf{z}}_{k\|k-1} + \mathbf{W}_k(\mathbf{y}_k - \hat{\mathbf{y}}_{k\|k-1})$
$P_{k\|k} = P_{k\|k-1} - W_k P_{yy,k\|k-1} W_k^T$
Fixe-Lag Smoothing
$\hat{\mathbf{x}}_k^{final} = \hat{\mathbf{x}}_{k\|k+N+L-2}$

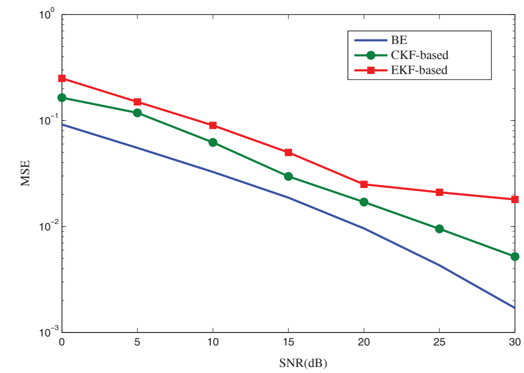

Fig. 1: Mean square error of channel tracking for $f_D T = 0.01$.

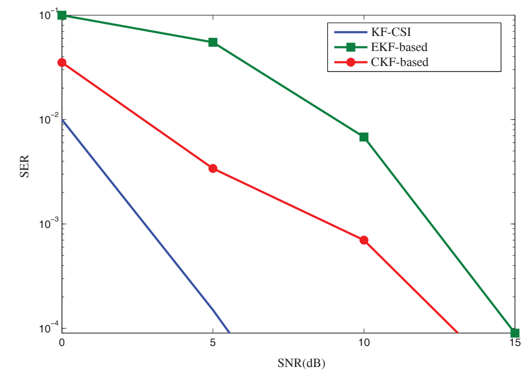

Fig. 2: Symbol error rate for detected symbols for $f_D T = 0.01$.

The symbol error rate for detected symbols using the CKF, the EKF and a Kalman equalizer with perfect channel knowledge (KF-CSI) is presented in Fig. 2. The difference between the CKF and the KF-CSI is of approximately 7dB, while the difference between the EKF and the KF-CSI is of approximately 9dB. Analyzing the SER values, the CKF once again outperforms the EKF.

As previously discussed, the employment of a first-order autoregressive process to model the channel coefficients dynamic behavior results in performance degradation. To illustrate the modeling error impact and the results achieved by the proposed algorithm with aging factor when subjected to a "real" fading channel, we set a scenario with 2 transmitting and 4 receiving antennas sending the QPSK symbols in a pilot based scheme. In such scheme, one known training symbol is transmitted, followed by 5 data symbols. The channel coefficients were generated by a 2-path Rayleigh channel with 0 dB average power and a normalized Doppler rate of $f_D T = 0.005$, according to Jake's model [9].

Fig. 3 depicts a comparison between the MSE of the channel estimation of standard CKF and a EKF-based receivers, and a

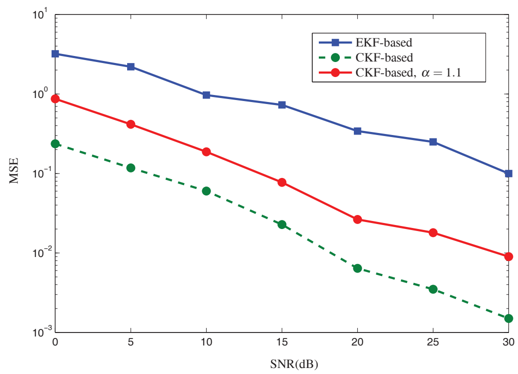

Fig. 3: Mean square error of channel tracking for $f_D T = 0.005$.

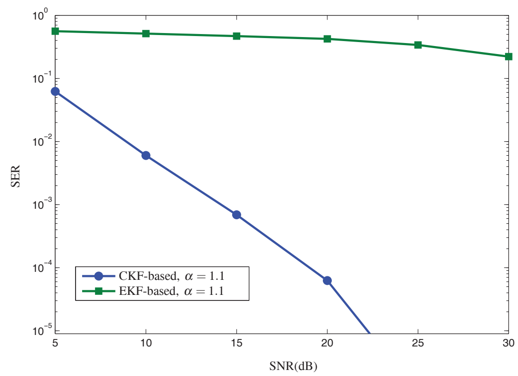

Fig. 4: Symbol error rate for detected symbols using aging factor.

CKF-based receiver with an aging factor of $\alpha = 1.1$. Note that the proposed algorithm with aging factor has the best estimation performance when compared to the receivers that do not take the modeling errors under consideration. Also, note that the EKF-based receiver presents a much poorer estimation when compared to the CKF-based algorithms, showing that modeling mismatches have a deeper impact on the *extend Kalman filter* estimation ability, mainly due to its first-order linearization.

The symbol error rates for receivers using the CKF and the EKF with aging factor are shown in Fig. 4. The SER resulting from the EKF is almost constant for the whole SNR range considered, indicating that it is not able to overcome modeling errors. The filter based on the CKF, on the other hand, presents low error rates, and the SER decreases with the increase of the SNR. This result shows that the CKF inspired algorithm is able to track a "real" fast frequency-selective fading channel, in spite of the fact that it models the channel dynamics as a first-order AR process.

V. Conclusions

In this paper, the *cubature Kalman filter* (CKF), was used to perform joint channel estimation and data detection in fast frequency-selective MIMO environments. In order to jointly estimate the channel and detect the symbols, the problem was modeled in the state-space form by defining an extended state-equation containing both the channel coefficients and the transmitted symbols. This formulation leads to a nonlinear observation equation, which prevents the use of the well-known Kalman Filter.

Once the estimation problem was defined, the *cubature Kalman filter* (CKF) was presented, along with two others techniques used to improve the receiver's estimation and detection ability: smoothing and fading memory. As a result, a semi-blind algorithm based on the *cubature Kalman filter* is proposed that efficiently solve the joint channel estimation and data detection problem.

Simulations indicate the ability of the CKF-based filter to efficiently track fast channel variations and correctly detect symbols with a low error rate. One advantage of the proposed receiver is that the computational complexity of the CKF grows only linearly with the dimension of the state and is less susceptible to numerical issues than other methods. The proposed algorithm outperforms the *extended Kalman filter* (EKF), which is commonly used for nonlinear filtering problems, presenting a smaller MSE for channel estimation and a smaller SER for symbol detection than the EKF. Furthermore, by basing its estimates in more recent measurements, the proposed CKF-based algorithm is capable of dealing with modeling errors, being able to track "real" fast frequency-selective fading channels.

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