THE CAPACITY OF BINDERS FOR MIMO DIGITAL SUBSCRIBER LINES

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ABSTRACT

Crosstalk is one of the main limiting factors in the data rates achievable by digital subscriber line (DSL) systems, and several algorithms have been proposed to mitigate this impairment. In this paper, we compare the capacity of binders under different crosstalk-mitigating techniques. When computing capacity, we also compare two different power constraints: either on the total power in the binder or on the power in each pair. We will see that, for the scenarios considered in this paper, the fact that the signals are jointly processed in one or both ends of the DSL link leads to roughly the same performance, which is far superior to that of systems with no cooperation between the users. Both power constraints also lead to similar achievable rates.

I. INTRODUCTION

Digital subscriber line (DSL) systems exploit the twisted pairs traditionally used for phone services to transmit high-rate data services such as internet access. Twisted pairs from different users are normally grouped in binders, which may contain as many as 50 pairs. Due to the proximity of these pairs, their signals are magnetically coupled, generating crosstalk between the users. Crosstalk is one of the main impairments of DSL systems, and one of the main factors limiting the achievable data rates of these systems.

Currently deployed digital subscribe line (DSL) systems use a single wire pair to transmit data between the central office (CO) and the end user. Furthermore, the users are not processed jointly at the CO. In other words, current DSL systems essentially transmit through single-input single-output (SISO) channels. In these cases, little can be done to mitigate crosstalk; perhaps the only choice is to allocate spectrum to different users so as to minimize the impact of crosstalk.

It has long been recognized that jointly processing the signals from different users may improve performance of DSL systems [1], [2]. These works exploit the fact that the CO has access to the signals of all the pairs in a binder to mitigate crosstalk using signal processing techniques, achieving rates close to the optimum. Note that [1], [2] assume that the signals are jointly processed only at the CO side. In fact, these works assume that, at the costumer premises (CP), users have access to a single pair, and thus may process only its own signal.

However, users often have access to more than one pair. This if fairly common in Europe, and may also occur in connections between central offices and remote terminals. In these cases, we have a multiple-input and multiple-output (MIMO) DSL system. In a MIMO system, the signals in the wire-pairs may be jointly processed at both ends of the links, enabling the use of crosstalk-mitigating techniques that are even more powerful than those proposed in [1], [2].

In this paper, we compute the capacity of several binders based on different signal processing algorithms. When computing the capacity, we also compare two different power constraints. First we use the traditional waterfilling constraint, which limits the total power transmitted in the binder. We also use an alternative constraint of more practical interest: a per-pair power constraint. This limitation reflects the fact that the signal in each pair must go through a power amplifier, whose output power is limited by practical considerations.

II. CHANNEL MODEL AND PROBLEM STATEMENT

In this paper, we consider an ideal MIMO-DSL system, in which all the lines in a binder can be jointly processed both at the receiver and at the transmitter. We assume that frequency-division duplexing is used, so that there is no near-end crosstalk. Furthermore, as with all DSL systems, the use of orthogonal-frequency division multiplexing transforms the time-dispersive channel into multiple parallel channels, called subchannels or tones, with no intersymbol-interference. In this case, the channel output at a given tone, $\mathbf{y}$ is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k, \tag{1}$$

where $\mathbf{x}_k$ is a vector containing the signals transmitted from each pair, $\mathbf{H}_k$ is a matrix that models the crosstalk between the lines and $\mathbf{n}_k$ represents the additive white Gaussian noise (AWGN) with covariance matrix $\sigma_k^2 \mathbf{I}_N$.

Several attempts were made to mathematically model the matrix $\mathbf{H}_k$ on each tone [3]. In this paper, we employ what is perhaps the most accurate model in the literature, the one developed by Bin Lee in his doctoral thesis [4]. Based on Bin Lee's model, the aim of this paper is to compare the capacity achieved by systems employing different types of crosstalk mitigating techniques and different spectrum allocation algorithms. We will also use these algorithms to determine the capacity of an actual binder whose channel matrix was measured.

III. CROSSTALK-MITIGATING ALGORITHMS

In this section, we describe the three signal processing algorithms used in this paper to mitigate crosstalk. The first one attempts no cancelation whatsoever, and is termed no signal processing (NSP). The second is based on the inversion of the channel matrix, and is called zero-forcing (ZF), since it forces the crosstalk to zero. The third is based on the singular-value decomposition (SVD) of the channel matrix.

Note that the techniques used in this paper assume perfect channel knowledge. Furthermore, all these techniques can be seen as the multiplication of the transmitted signal by a preprocessing matrix before transmission and/or the multiplication of the received signal by a receive matrix. Thus, all schemes can be represented as in Fig. 1. In this figure, N represents the number of pairs in the binder, $\tilde{x}_k^n$ is the information symbol transmitted in the k-th tone of the n-th user, x_k^n (y_k^n) is the signal actually transmitted (received) in the k-th tone of the n-th pair, and $\tilde{y}_k^n$ is the received signal in the k-th tone of the n-th user. Note that this linear transceiver scheme creates a virtual channel between the transmitted and received signals for the users:

$$\tilde{\mathbf{y}}_k = \mathbf{R}_k \mathbf{H}_k \mathbf{P}_k \tilde{\mathbf{x}}_k + \tilde{\mathbf{n}}_k, \tag{2}$$

where $\tilde{\mathbf{n}}_k$ represents zero-mean additive Gaussian noise with covariance matrix $\mathbf{C}_{\tilde{\mathbf{n}}_k} = \mathbf{R}_k \mathbf{R}_k^H$.

III-A. No Signal Processing

In this case, we assume that there is no cooperation between the users at either end of the DSL link. In other

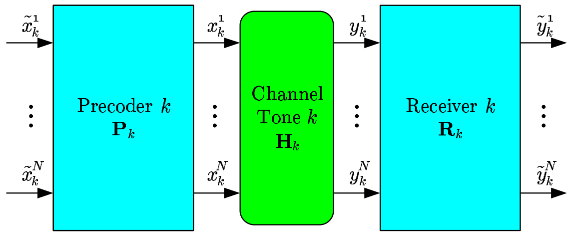


Fig. 1. Tone k of a DSL system with a general linear transceiver.

words, $\tilde{y}_k^n$ depends only on y_k^n, and x_k^n depends only on $\tilde{x}_k^n$. As a consequence, the receiver and preprocessing matrices for all tones must be diagonal, and no attempt can be made to mitigate crosstalk. On the other hand, the receiver may compensate for the direct channel attenuation. To that end, we make the n, n entry of $\mathbf{R}_k$ equal to the inverse of the channel attenuation for the n-th pair at the tone, i.e.,

$$[\mathbf{R}_k]_{n,n} = ([\mathbf{H}_k]_{n,n})^{-1}. \tag{3}$$

Also, we assume that the preprocessing matrix in Fig. 1 is the identity matrix for all tones.

III-B. Zero Forcing

The ZF algorithm assumes that the signals are jointly processed only at the CO, and that a single pair is available to each user. Now, in the downstream the CO is the transmitter, while it is the receiver in the upstream. Thus, for downstream transmission we can only perform ZF precoding, while ZF processing at the receiver is all that is available for upstream transmission. In both cases, following [2], we use the inverse of the channel matrix for processing, so that the equivalent channel in (2) is the identity matrix.

A well-known drawback of ZF techniques is power enhancement. In the downstream, a ZF precoder is used, which causes an SNR penalty by artificially increasing the power transmitted in each line. This power penalty is taken into account in the power allocation algorithms presented in the next section. On the other hand, in the upstream, the use of a ZF receiver causes noise enhancement, since the equivalent noise in (2) now has a covariance matrix given by $\mathbf{H}_k^{-1}\mathbf{H}_k^{-H}$. This SNR penalty is easily taken into account in the power allocation algorithms. Both upstream and downstream penalties may lead to a capacity loss. However, as we will see, the capacity of ZF schemes is similar to that of other methods, implying that both types of power enhancement have negligible effect.

III-C. Singular-Value Decomposition

In the fully-coordinated case, both the CO and CP can jointly process all the DSL pairs. This case corresponds

Table I. MIMO Transceiver Structures - Downstream

	SVD	ZF	NSP
$\mathbf{P}_k$	$\mathbf{V}_k$	$\mathbf{H}_k^{-1}$	$\mathbf{I}_N$
$\mathbf{R}_k$	$\mathbf{U}_k^H$	$\mathbf{I}_N$	$\text{diag}\left(\mathbf{H}_k^{-1}\right)$
$\mathbf{C}_{\tilde{\mathbf{n}}}$	$\sigma_k^2 \mathbf{I}_N$	$\sigma_k^2 \mathbf{I}_N$	$\text{diag}\left(\mathbf{H}_k^{-1}\right)^2$
g_k^n	$\left\|\left[\mathbf{\Sigma}_k\right]_{n,n}\right\|^2$	1	1

Table II. MIMO Transceiver Structures - Upstream

	SVD	ZF	NSP
$\mathbf{P}_k$	$\mathbf{V}_k$	$\mathbf{I}_N$	$\mathbf{I}_N$
$\mathbf{R}_k$	$\mathbf{U}_k^H$	$\mathbf{H}_k^{-1}$	$\text{diag}\left(\mathbf{H}_k^{-1}\right)$
$\mathbf{C}_{\tilde{\mathbf{n}}}$	$\sigma_k^2 \mathbf{I}_N$	$\sigma_k^2 \left\{\mathbf{H}_k^{-1}\mathbf{H}_k^{-H}\right\}$	$\text{diag}\left(\mathbf{H}_k^{-1}\right)^2$
g_k^n	$\left\|\left[\mathbf{\Sigma}_k\right]_{n,n}\right\|^2$	1	1

to a single-user MIMO case, where the CO communicates with a single-user CP, both of them equipped with as many modems as used pairs. In this fully-coordinated case, an SVD-based solution was implemented. We now describe this solution.

Let $\mathbf{H}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$ be the SVD of the channel matrix $\mathbf{H}_k$. Then, the precoding and receiver matrices are given by $\mathbf{V}_k$ and $\mathbf{U}_k^H$, respectively, yielding an equivalent channel given by $\mathbf{\Sigma}_k$. Since this is a diagonal matrix, we see that crosstalk has been eliminated.

It is interesting to notice that the receiver matrix is unitary, so that it does not change the noise statistics, i.e., $\mathbf{C}_{\tilde{\mathbf{n}}_k} = \sigma_k^2 \mathbf{I}_N$. The precoding matrix is also unitary, which implies that the total power (sum of the power in all pairs) before and after precoding are the same. In other words, the SVD effectively decouples all the users, with no noise enhancement and no increase in the total power. This observation leads to a simple power allocation algorithm, the well-known waterfilling solution described in the next section. On the other hand, the power in a specific pair before and after precoding are different. Thus, if there is a per-line power contraint, the precoding operation must be accounted for. This can be done with the optimal power allocation algorithm described in the next section.

III-D. Summary

The signal processing algorithms described in this section are summarized in table I for downstream transmission and in table II for the upstream. In these tables, the entry g_k^n corresponds to the magnitude-squared of the gain of the equivalent direct channel in (2) for the n-th pair and the k-th tone, i.e.,

$$g_k^n = |[\mathbf{R}_k \mathbf{H}_k \mathbf{P}_k]_{n,n}|^2. \tag{4}$$

This value will be used by the power allocation algorithms in the next section.

IV. POWER ALLOCATION ALGORITHMS

Traditionally, optimal power allocation is determined under the assumption that the total power to be transmitted is limited. In the MIMO-DSL case, this is equivalent to constraining the sum of the powers in all the pairs in a binder. However, it may be of practical interest to actually consider a power constraint per line. This is due to the power limitation at the amplifier of each modem, as well as restrictions to avoid excessive interference with other systems. We begin this section with a brief presentation of the waterfilling solution [3], which achieves the channel capacity when the total transmit power is limited. Then, we present the optimal power allocation algorithm [2], which determines the optimal power allocation for a precoded system when there is a per-line power constraint.

IV-A. Waterfilling

As mentioned in section III-C, the use of an SVD-based transceiver yields several parallel and independent AWGN channels. In this case, the noise variance and the total power transmitted in both the equivalent and the actual channels are the same. The problem of power allocation in this case is standard, and its solution is the well-known waterfilling solution [3]: the power allocated to the information symbol $\tilde{x}_k^n$ is given by

$$\tilde{s}_k^n = \left[\lambda - \Gamma \frac{\sigma_k^2}{g_k^n}\right]^+, \tag{5}$$

where

$$[x]^+ \triangleq \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}, \tag{6}$$

and where Γ stems from the use of the gap approximation [3]. In (5), λ represents the water level, which must be chosen so that the total transmit power is equal to the maximum value allowed, P. In other words, λ is chosen so that

$$\sum_n \sum_k \tilde{s}_k^n = P. \tag{7}$$

Finally, it should be pointed out that $\tilde{s}_k^n$ is the power allocated to the symbols at the k-th tone of the n-th user *before* the precoding matrix (see Fig. 1).

IV-B. Optimal Power Allocation with Preprocessing

In this section, we describe a power allocation algorithm for a system based on a linear preprocessing algorithm. This algorithm, known as optimal power allocation (OPA)

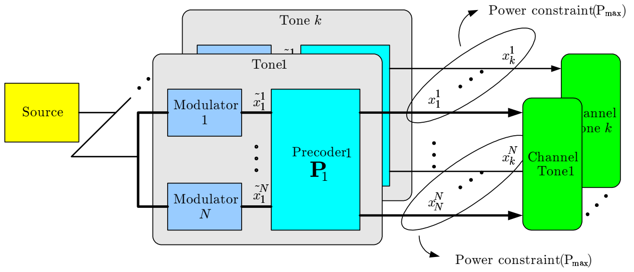


Fig. 2. Detail of the precoding matrix and the power constraint per line.

and presented in [2], limits the power transmitted on each line.

Consider the precoding operation

$$\mathbf{x}_k = \mathbf{P}_k \tilde{\mathbf{x}}_k, \tag{8}$$

shown in detail in Figure 2. The power of each information symbol $\tilde{x}_k^n$ is denoted by $\tilde{s}_k^n$ and the power of the precoded symbols x_k^n is denoted by s_k^n. The optimal power allocation problem can be then stated as follows

$$\{\tilde{s^*}_k^n\}_{k=1\ldots K}^{n=1\ldots N} = \arg \max_{\tilde{s}_k^n,\ \forall k,n} \sum_n \sum_k b_k^n\left(\tilde{s}_k^n\right) \tag{9}$$

subject to

$$\begin{aligned} &\sum_k s_k^n \leq P, \quad \forall n \\ &s_k^n \geq 0, \quad \forall n,k\ , \end{aligned} \tag{10}$$

where $b_k^n\left(\tilde{s}_k^n\right)$ is the maximum bit rate that can be transmitted using power $\tilde{s}_k^n$.

Using the gap approximation [3] and a diagonalyzing precoder such as the ZF or the SVD, the capacity in (9) is given by

$$b_k^n\left(\tilde{s}_k^n\right) = \log_2\left(1 + \frac{g_k^n \tilde{s}_k^n}{\tilde{\sigma}_k^2 \Gamma}\right) \tag{11}$$

where g_k^n is the square of overall gain between transmitted symbol $\tilde{x}_k^n$ and the received symbol y_k^n, Γ is the SNR gap, and $\tilde{\sigma}_k^2$ is the noise variance at tone k after receiver processing (refer to Tables I and II).

Also, considering the elements of the preprocessing matrix $p_k^{n,m} \triangleq [\mathbf{P}_k]_{n,m}$, the output power for each modem is given by

$$s_k^n = \sum_m |p_k^{n,m}|^2\, \tilde{s}_k^m\ . \tag{12}$$

Combining both equations, the original optimization problem becomes

$$\{\tilde{s^*}_k^n\}_{k=1\ldots K}^{n=1\ldots N} = \arg \max_{\tilde{s}_k^n,\ \forall k,n} \sum_n \sum_k \log_2\left(1 + \frac{g_k^n \tilde{s}_k^n}{\tilde{\sigma}_k^2 \Gamma}\right) \tag{13}$$

Table III. Optimal Power Allocation with Precoding

repeat

$$\forall n,k:\ \tilde{s}_k^n = \left[\frac{1}{\sum_m \lambda_m |p_k^{m,n}|^2} - \Gamma\frac{\tilde{\sigma}_k^2}{g_k^n}\right]^+$$

$$\forall n,k:\ \lambda_n = \left[\lambda_n + \mu\left(\sum_k \sum_m |p_k^{n,m}|^2\, \tilde{s}_k^m - P\right)\right]^+$$

until convergence

subject to

$$\begin{aligned} &\sum_k \sum_m |p_k^{n,m}|^2\, \tilde{s}_k^m \leq P, \quad \forall n \\ &s_k^n \geq 0, \quad \forall n,k\ . \end{aligned} \tag{14}$$

The solution of the above optimization problem can be found by using Lagrange multipliers. In [5], Cendrillon proposes the iterative algorithm shown in Table III to find the optimum solution.

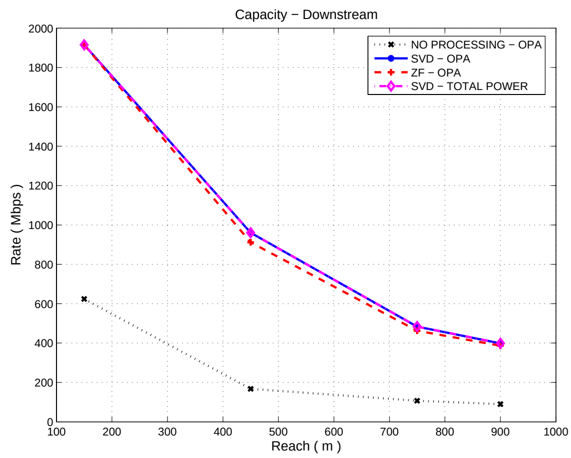
V. SIMULATIONS

In this section we present simulation results that compare the capacity achieved by the different transceivers and power allocation algorithms described in the previous section. In particular, we computed the capacity of SVD, ZF and no processing transceivers with OPA, and of an SVD transceiver with total power constraint, both for upstream and downstream channels. In the sequel, we describe the two scenarios simulated and present the capacity results.

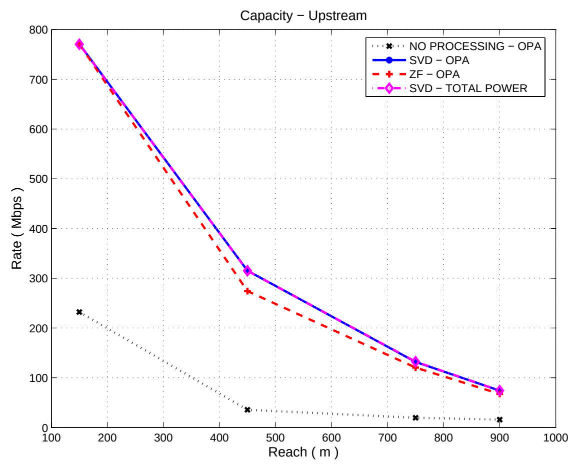
V-A. First Simulation: results for the Bin Lee's channel model on MIMO-VDSL

In this simulation, the rate as a function of the reach was evaluated for a MIMO-VDSL channel generated using Bin Lee's model. We used a binder with 10 VDSL pairs, with 4096 tones. The wire diameter is 0.5 mm (24-AWG). The target bit error rate (BER) was set to 10^{-7} or less, the coding gain was set to 3.8 dB and the noise margin to 6 dB, which leads to an effective gap of $\Gamma = 9.8 - 3.8 + 6 = 12$ dB. In accordance with the VDSL standards [6], [7], the tone spacing Δf is set to 4.3125 kHz and the DMT symbol rate f_s to 4 kHz. Background noise is composed of white thermal noise with a PSD of -140 dBm/Hz and the available transmitted power for each line is +11 dBm.

Figure 3 depicts the rates achieved by the three processing algorithms implemented (no processing, ZF and SVD) using OPA, as well as an SVD algorithm using total power constraint. The results are plotted as a function of the reach. One can see that the SVD performance is a little better than the ZF. Also, there is no difference between the SVD results obtained using a per-line power constraint (OPA) and a total binder power constraint. This is because the direct channel and the crosstalk for all the ten pairs is almost the same, which leads to a similar SNR value in all the cables. In other words, the channels for all pairs are similar, so there is no reason to allocate more power to one line than to another. Finally, we observe that all techniques outperform by far the results obtained without any signal processing, especially for short distances.



(a) Downstream Rate.



(b) Upstream Rate.

Fig. 3. Rate as a function of reach for no processing, SVD and ZF algorithms using OPA power allocation and SVD using total power.

V-B. Second Simulation: MIMO-ADSL2+ results for the measured channel

The measured channel data was generated for a MIMO-ADSL2+ binder with 10 pairs for two distances: 500 meters and 1500 meters. The capacity of this binder was evaluated for the following signal processing algorithms: no signal processing, SVD and ZF. To highlight the effect of noise in the algorithms, we measured the capacity as a function of the noise power. As per the ADSL2+ standard [8], 512 tones were used, the tone spacing Δf is set to 4.3125 kHz and the DMT symbol rate f_s to 4 kHz. Background noise is composed of white thermal noise and the available transmitted power for each line is +11 dBm. Our computations are based on the gap approximation: the target bit error rate (BER) is 10^{-7} or less, the coding gain was set to 3.8 dB and the noise margin to 6 dB, which leads to an effective gap of $\Gamma = 9.8 - 3.8 + 6 = 12$ dB.

In Figure 4 we can notice that, for a large range of noise power, using signal processing algorithms like SVD and ZF produces the same data rate as using no crosstalk-mitigating algorithm. The similar performance for all algorithms can be explained by the fact that the channel matrices for all tones are almost diagonal, since the crosstalk is very small for the ADSL2+ frequency range. In fact, the only difference in performance happens for the shorter binder at low noise levels, where the crosstalk become the dominant impairment. We suppose that a difference would also be observed for the longer cable, but for a noise level below the range we considered. Notice that the difference in the upstream transmission is even less pronounced than in downstream.

VI. CONCLUSIONS

In this paper, we have computed the capacity as a function of the distance for binders of twisted pairs, as used in DSL systems. We have used several signal processing and power allocation algorithms. We have shown that crosstalk mitigation techniques may yield significant gains over systems that do not attempt to cancel crosstalk. However, our simulation results suggest that the capacity of a system that jointly processes signals at both ends of the communications link is similar to the capacity of a system that performs joint processing only at the CO side. Finally, the results in this paper also indicate that imposing a per-line power restriction incurs almost no capacity penalty under the simulated scenarios. This is because all the pairs see similar channels, so that there is no reason to allocate more power to one pair than to another. In other words, waterfilling allocates similar powers to all the pairs, so that it approximately observes a per-line power restriction.

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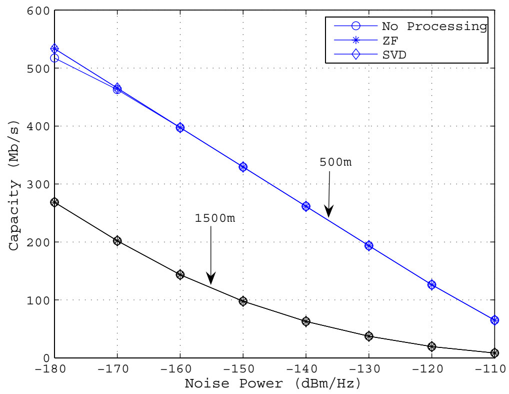
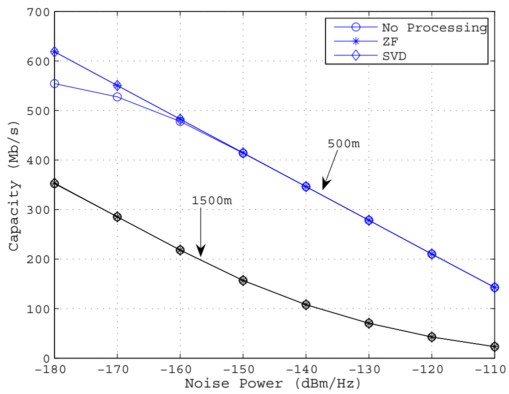


(a) Upstream Rate.



(b) Downstream Rate.

Fig. 4. Rate as a function of noise power for no processing, SVD and ZF algorithms using OPA for the MIMO-ADSL2+ measured channels.

- MUSE and the Eureka - Celtic BANITS projects respectively.

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