Blind turbo receivers with fast least-squares channel estimation and soft-feedback equalisation

M.B. Loiola, R.R. Lopes and J.M.T. Romano

Low-complexity blind turbo receivers composed of a soft-feedback equaliser (SFE) and fast least-squares (FLS) channel estimators are proposed. To reduce the complexity of the SFE computation, a function approximation is proposed instead of numerical algorithms in a specific step of the equaliser's computation. Concerning FLS channel estimators, a specific filter parameters initialisation procedure is proposed in each turbo iteration to avoid possible numerical instabilities. Estimation of noise variance is also considered. The proposed scheme can perform as the turbo equaliser with perfect channel knowledge from a certain signal-to-noise ratio.

Introduction: One of the most effective methods to mitigate the effects of intersymbol interference (ISI) in communication systems is turbo equalisation [1, 2]. The majority of turbo equalisers proposed in the literature, such as those presented in [1, 2], assumes perfect knowledge of channel coefficients and noise power. However, this information is not normally available to the receivers and thus channel estimators are fundamental to those turbo equalisers. In the turbo receivers proposed in [3–6], channel estimators are included in the feedback loop of turbo equalisers, taking advantage of the soft decisions provided by the decoder to refine channel estimates in each turbo iteration. Specifically, in [3] a least mean square (LMS)-based algorithm is used for channel estimation, while in [4] both LMS and recursive least-squares algorithms are employed. A modified Kaiman filter is proposed in [5] and a linear complexity approximated minimum mean squared-error (MMSE) channel estimator using Gaussian message passing is developed in [6]. However, none of these solutions combines the convergence speed of Kaiman or RLS algorithms with the low complexity of LMS-based algorithms. Hence, in this Letter we propose blind turbo receivers composed of the linear-complexity soft-feedback equaliser (SFE) [1] and linear complexity, MMSE channel estimators using fast least-squares (FLS) algorithms [7–9]. Thanks to the block processing inherent to turbo receivers and to a judicious choice of filter parameters, we overcome the numerical instabilities of FLS algorithms. It is also worth noting that the turbo receivers presented in [1–6] assume perfect knowledge of noise power. In this Letter, however, noise power is not known *a priori* and is also estimated in each turbo iteration. We further simplify the computation of SFE coefficients.

Turbo equalisation with soft-feedback equaliser: The SFE [1] is a low-complexity structure, similar to a decision-feedback equaliser, which combines linear equalisation and soft ISI cancellation and where coefficients are chosen to minimise the mean squared-error (MSE) between the equaliser output and the transmitted sequence. The SFE combines the extrinsic equaliser outputs and *a priori* information to form more reliable estimates of the residual postcursor ISI. Also, by adopting a Gaussian model for the equaliser outputs and *a priori* information, the MMSE equaliser becomes linear-complexity and time-invariant. As shown in [1], the equaliser coefficients depend on the quality of both equaliser outputs and *a priori* information through the expression $\Psi_1(\gamma) = \mathrm{E}\,[\tanh\,(u/2)]$, where $u \sim \mathcal{N}(\gamma, 2\gamma)$ and γ is related to the quality of *a posteriori* probabilities. Since there is no closed-form formula for Ψ_1 (7), the authors of [1] used numerical algorithms to compute $\Psi_1(\gamma)$. In this Letter, we propose the following approximation, based on [10], to the computation of $\Psi_1(\gamma)$:

$$\Psi_1(\gamma) = \begin{cases} 0.4808\gamma + 10^{-4}, & \gamma < 0.2 \\ 1 - \exp(0.0218 - 0.4527\gamma^{0.86}), & \gamma \geq 0.2 \end{cases} \tag{1}$$

The values in (1) were optimised to minimise the maximum absolute error between (1) and $\Psi_1(\gamma)$. We verified by computer simulations that the use of (1) instead of numerical algorithms incurs no performance degradation of the SFE.

Blind turbo receiver: The blind turbo receiver considered in this work is shown in Fig. 1.

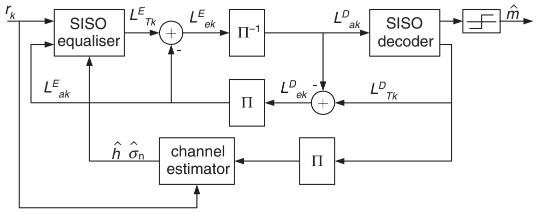

Fig. 1 *Turbo receiver*

To allow usual soft-input, soft-output (SISO) equalisers to work blindly, we need to provide them with initial estimates of channel coefficients and noise variance. Thus, we suppose that the channel length μ is known and that, for each received block, the initial channel estimate $\hat{\boldsymbol{h}}_o$ has only one non-zero element, which is given by $\hat{\sigma}_{n,0}$, where

$$\hat{\sigma}^2_{n,0} = \frac{1}{2N}\sum_{k=0}^{N-1}|r_k|^2 \tag{2}$$

is the initial estimate of noise variance, r_k is the received signal at instant k, and N is the block length. The initial conditions estimate the SNR is 0 dB, while maintaining $\hat{\boldsymbol{h}}_o$ and $\hat{\sigma}_{n,0}$ consistent with the received energy. The initial estimates in (2) are recomputed for every received block and are used by the SISO equaliser in the first turbo iteration. Also, in the first turbo iteration of each block, the channel estimator initialises its update procedure with (2). In subsequent iterations, this initialisation uses channel and noise variance estimates computed in the previous iteration.

To estimate the channel coefficients we propose the use of FLS algorithms, such as the fast Kaiman (FK) [7], the fast *a posteriori* error sequential technique (FAEST) [8] and the fast transversal filter (FTF) [9]. These algorithms are mathematically equivalent to the Kaiman filter, thus providing optimum MMSE estimates for linear systems embedded in Gaussian noise. Also, their complexity is linear in channel memory. In general, FLS algorithms can be described as composed by four transversal filters: a forward predictor $\boldsymbol{a}_k$, a backward predictor $\boldsymbol{b}_k$, a vector of channel estimates $\hat{\boldsymbol{h}}_k$ and a gain vector $\boldsymbol{g}_k$. All these filters are excited by the same input sequence, which is given by the soft symbol estimates computed from the *a posteriori* probabilities provided by the decoder. To overcome the numerical instabilities that can appear in such algorithms as time passes, we take advantage of the block processing inherent to turbo receivers and propose the following initialisation procedure: in the beginning of each turbo iteration, the filters $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{g}$ are set to zero, the likelihood variable [8] used in FAEST and FTF is set to one, and the energies of forward and backward prediction errors are set, respectively, to $E_{a,0} = E_0$ and $E_{b,0} = 1^{-\mu}E_0$, with $E_0 = 10^{-4}$. It is important to highlight that FLS algorithms can also diverge if channel and noise variance estimates are not set, respectively, to $\hat{\boldsymbol{h}}_0$ and $\hat{\sigma}_{0,n}$ in the beginning of the processing of each new block. Finally, to estimate the noise variance, we use the following estimator

$$\hat{\sigma}^2_{n,i} = \frac{1}{N}\sum_{k=0}^{N-1}|r_k - \hat{\boldsymbol{x}}_k^\top\hat{\boldsymbol{h}}_k|^2 \tag{3}$$

where $\hat{\boldsymbol{x}}$ is a vector containing the hard symbol estimates computed from the *a posteriori* probabilities provided by the decoder and i represents the ith turbo iteration.

Simulation results: To analyse the performance of the proposed linear-complexity blind turbo receiver, we use the SFE as the SISO equaliser and the well-known BCJR algorithm [11] in the SISO decoder. To estimate the channel coefficients, we simulate LMS, FK, FAEST and FTF algorithms, since they all have computational complexities of the same order. Since FK, FAEST, FTF are mathematically equivalent, they present the same performance. Hence, we just show the curves for the turbo receiver with FAEST channel estimator with a unitary forgetting factor. We consider random interleavers, a recursive systematic convolutional code with generators (111, 101), BPSK modulation and a zero-mean additive white Gaussian noise with variance defined by the SNR. We also assume that the location of the highest energy channel tap is known and this location contains the only non-zero element of $\hat{\boldsymbol{h}}_0$. Wc note that, as stated in [1], the SFE can be extended to multilevel modulations using techniques such as those presented in [12].

We simulate two scenarios: in the first one, we transmit blocks of 512 information bits through the channel $0.5 + 0.71z^{-1} + 0.5z^{-2}$. We simulate 10 turbo iterations and the SFE has anti-causal and causal ISI cancelling filters of lengths 6 and 3, respectively. In the second scenario, we send blocks of 1024 information bits through the channel $0.227 + 0.46z^{-1} + 0.688z^{-2} + 0.46z^{-3} + 0.227z^{-4}$. We perform 15 turbo iterations and the SFE has anti-causal and causal ISI cancelling filters of lengths 9 and 4, respectively. The bit error rate (BER) curves are shown in Fig. 2. For comparison purposes, we also show in this Figure the performance of a turbo receiver using an SFE turbo equaliser with perfect channel state information (CSI). From Fig. 2, we observe that the blind turbo receivers with FLS channel estimators perform better than the receivers with LMS channel estimators, and that the FLS-based blind receivers reach the performance of turbo equalisers with perfect channel and noise variance knowledge from a certain SNR value.

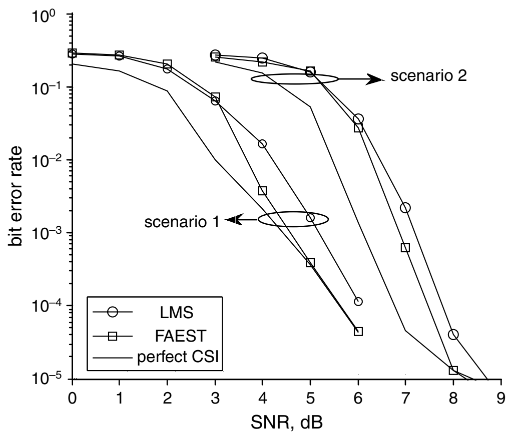

Fig. 2 *Bit error rate performances of proposed blind turbo receivers*

Conclusions: We propose low-complexity blind turbo receivers. The computation of SFE coefficients is simplified by an approximation to the Ψ_1 function. Also, to obtain linear-complexity MMSE channel estimates we use FLS algorithms. Thanks to the block processing characteristic of turbo receivers and to a judicious choice of channel estimator initialisation parameters, we avoid numerical instabilities that can possibly occur in FLS algorithms. By computer simulations we have verified that the proposed blind turbo receivers have a performance similar to the turbo receiver with perfect channel and noise power knowledge from a certain SNR. Hence, we conclude that good linear-complexity turbo receivers can be based upon the SFE equaliser and an FLS channel estimation algorithm.

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